Department of Education

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# First Steps in Mathematics

# Chance and Data

*Developing Probability and Statistics*

FIRST015 | First steps in Mathematics: Chance and data

First steps in Mathematics: Chance and data


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# Diagnostic Map: Chance and Data

## Measuring Chance

Students extend what they know about measuring *perceptual* attributes to measuring the *chance* of something happening.

In this way, it follows the same development as Measurement: students become aware of the attribute (Could it happen? Won't it happen? Is it likely?); they compare and order from more to less of that attribute (Which is more likely? Are they equally likely?); they measure the attribute by comparison with a unit (using numbers to say how likely), and then to the standard unit (placing events on a scale from 0 to 1).

However, probability is an abstract attribute and this makes measuring it more difficult for students to understand. As a result there is a chronological lag between a phase in the Measuring Chance Map and its equivalent phase in the Measurement Map.

Students may have the Number and Measurement understandings needed to be in or through a Measuring Chance phase, but will not be able to do what is expected unless they are provided with an appropriate curriculum in Chance measurement.

## Data Management

To collect, process and interpret data, students will need to draw on their understanding of Number and Measurement concepts. How they currently think in relation to Number and Measurement will influence how they respond to the activities provided for them, and hence what they are able to learn from them.

As a result, the phases outlined in the Measurement and Number Diagnostic Maps should be considered when interpreting students' responses to Data Management activities. Doing so will help in understanding why some students may struggle to achieve certain outcomes while others do not.

Below the information from the Number and Measurement Diagnostic Maps, there are some examples of what students will be in a position to do and understand as they move through the phases, given an appropriate curriculum in collecting, processing and interpreting data.

### Diagnostic Map Number: *Matching Phase*

Enter: 3–5 years Exit: 5–6 years

- Students use one-to-one relations to share and count out when asked: 'How many are there?' or 'How many are left?'

### Diagnostic Map Measurement: *Emergent Phase*

Enter: 2–3 years Exit: 5–7 years

Students initially attend to the overall appearance of size, recognising one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

#### Implications for Data Management

*By the end of these phases, students are in a position to*

- look at physical displays of familiar data and say which is most or has more
- sort and arrange data they have collected into familiar groupings
- count when asked to say how many in each group in a data display

*But, students*

- may count when asked to say how many in pre-arranged data, but focus on overall physical size rather than the numerical size of the group e.g., count 3 bananas and 5 strawberries, then say there are more bananas
- may focus on overall physical size rather than the numerical size of the group
- may say one group has more than another, but cannot say how much more

### Measuring Chance *Emergent*

Enter: 2–3 years Exit: 7–9 years

Students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language of their communities to describe likelihood.

As a result, they use this type of language themselves and describe familiar, easily understood events as being more or less likely, e.g., Mum said we *might* go to grandma's after school; we are *more likely* to go home than to grandma's; we *usually* go home after school.

*By the end of the Emergent phase, students typically*

- are beginning to show that they recognise an element of chance in many things that are a part of their lives
- understand expressions such as "will happen," "won't happen," and "might happen"
- are able to distinguish impossible events from events that are possible but unlikely

*But, as they enter the Matching and Comparing phase, they*

- distinguish between certain and uncertain events, but may not realise that certainty must also include events that are certain not to occur
- may be unable to distinguish equally likely events, e.g., may assume all colours are equally likely to appear when given a four-colour spinner with unequal sectors
- may understand that some things are more likely than others, but not be able to provide relevant reasons why events might be more or less likely to occur (e.g., believe they will spin a 6 because 6 is their favourite number)

### Diagnostic Map Number: *Quantifying Phase*

Enter: 5–6 years Exit: 6–9 years

- Students trust the count to describe quantity and interpret small numbers as compositions of other numbers.

### Diagnostic Map Measurement: *Matching and Comparing Phase*

Enter: 5–7 years Exit: 7–9 years

Students match in a conscious way in order to decide which is bigger by familiar readily perceived and distinguished attributes such as length, mass, capacity and time. They also repeat copies of objects, amounts and actions to decide how many fit (balance or match) a provided object or event.

As a result, they learn to directly compare things to decide which is longer, fatter, heavier, holds more or took longer. They also learn what people expect them to do in response to questions such as 'How long (tall, wide or heavy, much time, much does it hold)?' or when explicitly asked to measure something.

#### Implications for Data Management

*By the end of these phases, students are in a position to*

- suggest counting as a way of answering data questions that focus on comparing collections, e.g., will suggest counting our pets to answer the question "Which pets are more popular?"
- use skip counting to say how many in a tally
- suggest direct comparison when prompted to record growth data, e.g., we can cut a streamer to match the sunflower plant each week to see how much it grows
- use counting to help construct their data display, e.g., construct a block graph by counting how many in each group, then counting how many squares to colour in
- understand the need for a baseline and space blocks regularly to allow comparisons to be made
- place direct measurement data in sensible sequences using a baseline, e.g., cut paper strips to fit around their heads and make a bar (column) graph by lining up the bottom of strips
- choose to count to compare the sizes of groups, without prompting
- look at a bar graph and say which bar has more based on its length

*But, students*

- may not attend to equal units when grid lines are not provided, e.g., they may create the correct number of pictures for each group, but not use the same size for each picture
- cannot construct a scale on the vertical axis to represent frequencies or measurements (although they can use a common baseline and label the horizontal axis with the groups)
- may not realise that the relative lengths of the bars relate to quantities in the collected data
- may not use a scale on the axis to tell how many, instead preferring to count

## Measuring Chance *Matching and Comparing*

Enter: 7–9 years Exit: 9–11 years

Students draw on their experience to describe familiar things as more or less likely. They use expressions such as "very likely," "less likely," "equally likely," and "quite unlikely."

As a result, they are able to directly compare and order events from more to less likely and are able to justify their decision with relevant reasons.

*By the end of the Matching and Comparing phase, students typically*

- construct simple experiments and use counting to determine which event is more likely
- understand that certainty includes those events that must happen and those that cannot happen
- understand what it means for simple events to be equally likely, e.g., can see why a spinner with four equal sectors is equally likely to stop on any colour, and that one divided into unequal sectors will not
- can list all possibilities for straightforward situations when prompted

*But, as they enter the Quantifying phase, they*

- may be uncritically influenced by other dominant features when ordering objects by likelihood, e.g., may be influenced by personal preference or personal experience and so say, "It is less likely to rain tomorrow because it never rains on my birthday" or "I'm more likely to roll a 6 because I always roll a 6"
- may construct an experiment to determine likelihood, but be casual about ensuring fairness, even to the point of altering or fixing outcomes to produce the predicted results

## Diagnostic Map Number: *Partitioning Phase*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations and part-whole reasoning without having to see or visualise physical collections.

## Diagnostic Map Measurement: *Quantifying Phase*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

### Implications for Data Management

*By the end of these phases, students are in a position to*

- see that, when organising data, the categories can be reorganised without changing the overall total, e.g., to combine two brands of hamburgers, they need to add the two brands, or if they separate boys' and girls' responses for favourite take-out foods, the total must match the combined result
- see that they need to ensure that the data they collect is consistent
- produce and read pictographs or block graphs where each unit represents more than one piece of data
- produce simple two-way tables and Venn diagrams, partitioning totals between the cells or sections for straightforward data
- recognise that the length of bars in a bar graph can represent any numbers or measurements of data
- represent whole-number data in different ways, e.g., after measuring everyone's height, can produce a measurement graph that shows each child's height, or a frequency graph that shows the number of people of each height
- recognise that the length of bars and simple whole-number scales on the axis refer to quantities in the data collected, e.g., given a frequency graph about people's favourite take-out food, they know that the lengths of the bars represent the number of people naming each food in the survey
- use column and row headings to interpret what the numbers in simple two-way tables represent

*But, students*

- may be able to represent only whole numbers of units
- may not be able to work out how to represent the data when it is not a multiple of the unit
- may not realise that measurement data can be grouped
- may be unable to represent or read data using a continuous scale
- may not be able to convert to proportional measures to make comparisons
- may be unable to interpret the meaning between marked intervals on scales of frequencies or measures
- reading frequency data in two-way tables may not realise that to make sensible comparisons, the total frequencies need to be taken into account, e.g., students may say that more girls than boys like orange juice because 15 girls and 12 boys say they like it, but not realise that this information is misleading if there are 30 girls and 16 boys in the sample

## Measuring Chance *Quantifying*

Enter: 9–11 years Exit: 11–13 years

Students connect the idea of likeliness to the frequency of an event. They come to understand that numbers can be used to describe chance and that counting frequencies can provide a measure of likelihood for future events.

As a result, they trust information gained from repeated trials as an indicator of probability and are prepared to use this to order events and determine how likely they are.

*By the end of the Quantifying phase, students typically*

- draw on personal experience to compare and order a variety of chance-related events and order them along a continuum from events that cannot happen to events that must happen
- draw on numerical information alone to decide whether two simple events are or are not equally likely to occur
- are careful to ensure that an experiment is fair, e.g., that the sections of a spinner are equal or that the coin toss is consistent
- systematically list all possibilities, unprompted, to work out numerical probabilities for one-stage actions
- use experimental results and data about past events to determine a range of possible outcomes and informally use relative frequency to estimate probabilities

*But, as they enter the Measuring phase, they*

- may be unable to create devices such as spinners or bags of coloured balls to produce specified orders of probability, e.g., make a spinner which is most likely to come up red and equally likely to come up blue or green
- may compare the absolute number of occurrences to decide which is more likely, and not take account of the total number of trials, or consider all of the possibilities, e.g., might say that an event that occurs 8 out of 12 times is more likely to occur than an event that occurs 5 out of 7 because 8 is greater than 5
- may not realise that very many trials are needed to provide a reasonable estimate of probability, e.g., may think 30 trials is enough to test the fairness of a constructed spinner.

## Diagnostic Map Number: *Factoring Phase*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities and link equal groups and equal parts to fractions.

## Diagnostic Map Measurement: *Measuring Phase*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

### Implications for Data Management

*By the end of these phases, students are in a position to*

- think carefully about the accuracy of their data and recognise that data collection is about measuring different aspects of a situation
- understand that they can group measurement data in their display
- create axes that show discrete or continuous quantities, including time scales
- use simple proportional comparisons when interpreting data in tables and graphs, e.g., half as many people prefer pizzas to hamburgers; the height of the wheat is three times higher than it was at the end of week 2

*But, students*

- may not recognise when they need to convert their data to fractions or percentages to make sensible comparisons

## Measuring Chance *Measuring*

Enter: 11–13 years Exit: 15+ years

Students begin to quantify the chance of events occurring using proportional measures to say "how likely" or "how much more likely." They realise they can produce ratios by comparing the total number of occurrences to the total number of trials (experimental probability) or by comparing the number of desired outcomes to the number of possible outcomes (theoretical probability).

As a result, students understand that probability is the way we measure chance and that probability statements give a measure of how likely something is to happen using ratios, fractions, decimal fractions and percentages.

*By the end of the Measuring phase, students typically*

- use a range of information sources to put things in order from least likely to most likely, e.g., use research data or experimental data to form conclusions
- understand that the greater the number of trials, the greater its reliability as an indicator of likelihood
- use their understanding of equivalent fractions to judge equally likely events
- interpret the 0 to 1 scale in general usage and understand why the probability that a toss of a fair die will produce 5 is one-sixth
- identify all the outcomes for two- or three-stage situations, e.g., rolling two dice and tossing two or three coins
- are able to simulate a situation where it would be difficult, costly, or inappropriate to generate real data, by designing simple experiments that replicate a significant aspect of the situation, e.g., they use a die to simulate the colours of sweets in packets to explore the expected variation in the numbers of coloured sweets in each packet

*But, they*

- may not be able to overcome deep personal instincts about the probability an event should occur in spite of experimental or theoretical data, e.g., may believe that a head is more likely than a tail after a run of heads, or that a sequential set of Lotto numbers are less likely to be drawn than a given set of non-sequential numbers.
- may not recognise or trust calculations that would determine all possible outcomes for multiple-stage situations

## Diagnostic Map Number: *Operating Phase*

Enter: 11–13 years Exit: —

- Students can think of multiplication and division in terms of operators, and deal with proportional situations involving common and decimal fractions.

## Diagnostic Map Measurement: *Relating Phase*

Enter: 11–13 years

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students *work with measurement information* and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

### Implications for Data Management

*By the end of these phases, students are in a position to*

- choose from a wide range of measurement options when planning data investigations, using indirect measurements and creating measurement scales for non-standard attributes, e.g., they may create a scale from 1 to 5 to measure people's concern for environmental issues
- plan complex scales on axes to produce a wide range of graphs, including using class intervals, fractions, and percentages
- represent growth or change data over time by using a time scale, and approximate value within the intervals by joining the points when appropriate
- see that both axes can be made into number or measurement scales and used to show relationships between data, e.g., they could sketch a line graph to show how the amount of food on a plate varies over time at a buffet lunch as the plate is repeatedly filled with food, which is then eaten
- interpret displays showing relational information between measurements or frequencies
- interpret complex scales on graphs where not all scale markings are labelled

# Contents

## Foreword

The ***First Steps in Mathematics*** Resource Books and professional development program are designed to help teachers to plan, implement and evaluate the mathematics curriculum they provide for their students. The series describes the key mathematical ideas students need to understand in order to achieve the mathematics outcomes described in the Western Australian *Curriculum Framework* (1998).

Each Resource Book is based on five years of research by a team of teachers from the Department of Education and Training, and tertiary consultants led by Professor Sue Willis at Murdoch University. The ***First Steps in Mathematics*** project team conducted an extensive review of national and international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics.

Using tasks designed to replicate those in the research literature, team members interviewed students in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about major mathematical concepts. The Diagnostic Maps—which appear in the Resource Books for Number, Measurement, Space, and Chance and Data—describe these phases of development.

It has never been more important to teach mathematics well. Globalisation and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives. Teaching mathematics well to all students requires a high level of understanding of teaching and learning in mathematics and of mathematics itself. The ***First Steps in Mathematics*** series and professional development program will enhance teachers' capacity to decide how best to help all of their students achieve the mathematics outcomes.

The commitment and persistence of many teachers and officers of the Department of Education and Training, who contributed to the research and development of ***First Steps in Mathematics***, is acknowledged and appreciated. Their efforts have resulted in an outstanding resource for teachers. I commend this series to you.

**Paul Albert**
**Director General**
**Department of Education and Training**
**Western Australia**

CHAPTER 1

# What Are the Features of this Resource Book?

The ***First Steps in Mathematics: Chance and Data*** Resource Book will help teachers to diagnose, plan, implement and judge the effectiveness of the teaching and learning experiences they provide for their students.

This Resource Book includes the following elements.

- Diagnostic Map
- Mathematics Outcomes
- Markers of Progress
- Pointers
- Key Understandings
- Sample Learning Activities
- Sample Lessons
- 'Did You Know?' sections
- Background Notes

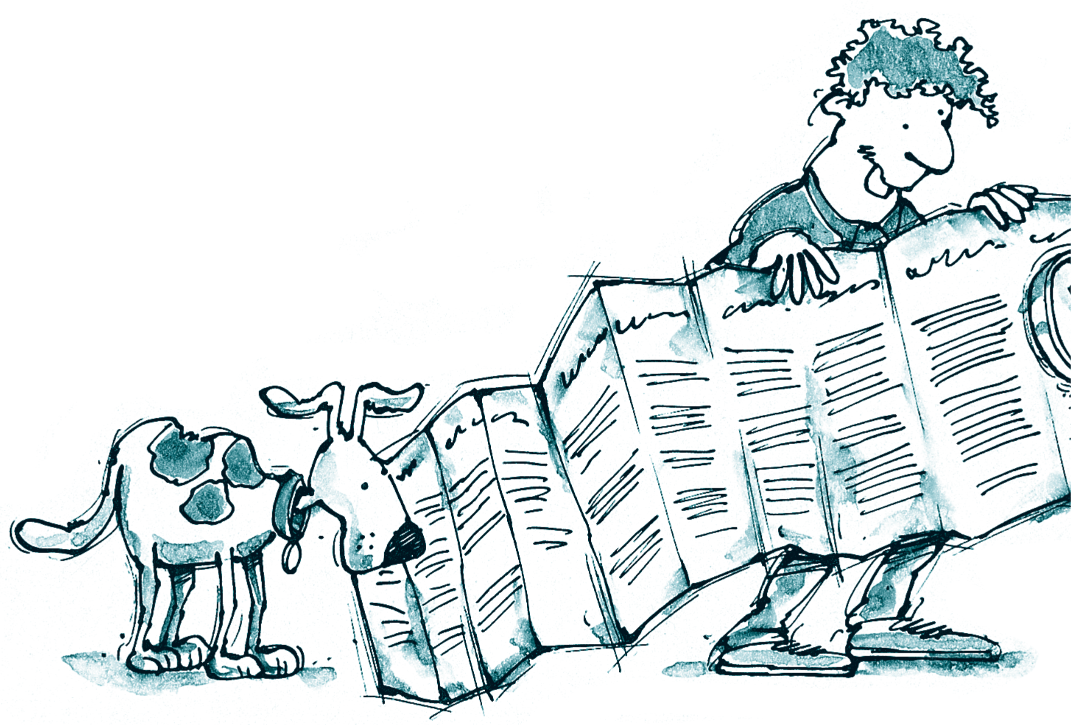
## Diagnostic Maps

Students' understanding of Chance and Data concepts is dependent on their current understanding of Number and Measurement concepts. Therefore, in the Chance and Data Resource Book, links are made to the Number and Measurement Diagnostic Maps.

The purpose of the Diagnostic Maps is to help teachers:

- understand why students seem to be able to do some things and not others
- realise why some students may be experiencing difficulty while others are not
- indicate the challenges students need to move their thinking forward, to refine their preconceptions, overcome any misconceptions, and so achieve the outcomes
- interpret their students' responses to activities.

Each map includes key indications and consequences of students' understanding and growth. This information is crucial for teachers making judgments about their students' level of understanding of mathematics. It enhances teachers' judgments about what to teach, to whom and when to teach it.



### *Using the Diagnostic Maps*

The Diagnostic Maps are intended to assist teachers as they plan their mathematics curriculum. The Diagnostic Maps describe the characteristic phases in the development of students' thinking about the major concepts in each set of outcomes.
The descriptions of the phases help teachers make judgments about students' understandings of
the mathematical concepts.

The text in the non-shaded sections of each map describes students' major preoccupations, or focus, *during* that phase of thinking about the mathematics strand.

The 'By the end' section of each phase provides examples of what students typically think and are able to do as a result of having worked through the phase.

The 'But as they enter' section illustrates that while students might have developed a range of important understandings as they passed through the phase, they might also have developed some unconventional or unhelpful ideas at the same time. Both of these sections of the Diagnostic Map are intended as a useful guide only.

## Mathematics Outcomes

The mathematics outcomes indicate what students are expected to know, understand and be able to do as a result of their learning experiences. The outcomes provide a framework for developing a mathematics curriculum that is taught to particular students in particular contexts. The outcomes for Chance and Data are located at the beginning of each section of the Resource Book.

## Markers of Progress

There are Markers of Progress outlined for each mathematics outcome. The ***First Steps in Mathematics*** Resource Books cover the typical achievement in primary school.

The Markers of Progress describe the development towards full achievement of the outcomes. Each student's achievement in mathematics can be monitored and success judged against the Markers of Progress.

As the phases of the Diagnostic Maps are developmental, and not age specific, the Markers of Progress will provide teachers with descriptions of the expected progress that students will make every 18–20 months when given access to an appropriate curriculum.

## Pointers

Each Marker of Progress has a series of Pointers. They provide examples of the skills students typically develop. The Pointers help clarify the meaning of the mathematics outcome and the differences between the Markers of Progress.

## Key Understandings

The Key Understandings are the cornerstone of the ***First Steps in Mathematics*** series. The Key Understandings:

- describe the mathematical ideas, or concepts, which students need to know in order to achieve the outcome
- explain how these mathematical ideas relate to the markers of progress for the mathematics outcomes
- suggest what experiences teachers should plan for students so they achieve the outcome
- provide a basis for the recognition and assessment of what students already know and still need to know in order to progress

- indicate the emphasis of the curriculum at particular stages
- provide content and pedagogic advice to assist with planning the curriculum at the classroom and whole-school levels.

The number of Key Understandings for each mathematics outcome varies according to the number of 'big mathematical ideas' students need to achieve the outcome.

## Sample Learning Activities

For each Key Understanding, there are Sample Learning Activities that teachers could use to develop the mathematical ideas of the Key Understanding. The activities are organised into three broad groups.

- Beginning activities are suitable for students 4 to 8 years old.
- Middle activities cater for students 8 to 10 years old.
- Later activities are designed for students 10 years and older.

If students in the later years have not had enough prior experience, then teachers may need to select and adapt activities from earlier groups.

## Sample Lessons

The Sample Lessons illustrate some of the ways in which teachers can use the Sample Learning Activities for the Beginning, Middle and Later groups. The emphasis is on how they can focus students' attention on the mathematics during the learning activity.

## 'Did You Know?' Sections

For some of the Key Understandings, there are 'Did You Know?' sections. These sections highlight common understandings and misunderstandings that students have. Some 'Did You Know?' sections also suggest diagnostic activities that teachers may wish to try with their students.

## Background Notes

The Background Notes supplement the information provided in the Key Understandings. These notes are designed to help teachers develop a more in-depth knowledge of what is required as students achieve the mathematics outcomes. (See pages 207–210.)

CHAPTER 2

# The Chance and Data Outcomes

The Chance and Data strand focuses on chance events and data-handling processes. In Chance, the focus is on developing students' ability to make predictions about how likely an event is in situations where there is uncertainty. In Data, the focus is on collecting, organising, analysing and presenting information (i.e. data).
As a result of their learning, students should be able to use their understanding of likelihood to compare and order everyday chance events, and be able to answer questions using data collection and/or interpretation.

During the primary years, students should:

- learn to recognise unpredictability in familiar daily activities and refine their use of the language of chance
- describe and order events from least to most likely
- begin to quantify 'how likely' it is that something will happen.

They should also begin to develop the understanding and skills needed to clarify the questions they want answered, and collect and handle data to help answer those questions. They should also consider such questions as 'What kind of data?' and 'How much do we need so we can feel reasonably confident in our conclusions?' The uncertainty involved in drawing conclusions from data is what connects 'chance' and 'data'. Thus, learning experiences should be provided that will enable students to understand chance, collect and organise data, summarise and represent data, and interpret data.

As a result of their learning experiences, students should be able to achieve the following outcomes.

#### Understand Chance

Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments and analysis.

#### Collect and Process Data

Plan and undertake data collections and organise, summarise and represent data for effective and valid interpretation and communication. There are two parts to this outcome—Part A: Collect and Organise Data and Part B: Summarise and Represent Data—with a separate chapter for each.

#### Interpret Data

Locate, interpret, analyse and draw conclusions from data, taking into account data collection techniques and chance processes involved.

## Integrating the Outcomes

The outcomes for Chance and Data are each dealt with in separate chapters of this book. This is to emphasise both the importance of each and the difference between them. For example, students need to learn to represent their data in graphs, tables or diagrams for others to read (Summarise and Represent Data), and to read, analyse and draw conclusions from data (Interpret Data). In the past, school work has often focused largely on the technical skills involved in producing tables, plots and graphs, and calculating averages. This has not helped students develop the ability to interpret data. This book pays separate and special attention to both these areas, which should help you to ensure that both receive sufficient attention, and that the important ideas about each are drawn out of the learning experiences you provide.

However, this does not mean that the ideas and skills underpinning each of the outcomes should be taught separately or that they will be learned separately. Indeed, the links between the outcomes are significant and inevitable. Consequently, many of the activities you provide should and will provide opportunities for students to develop their ideas about more than one of the outcomes. It is your role as teacher to ensure that the significant mathematical ideas are drawn from the learning activities so that students achieve each of the Chance and Data outcomes.

# A Snapshot of the Markers of Progress in Chance and Data

Students should not always be expected to be at the same level of understanding for each of the outcomes in Chance and Data. Students vary, so some may progress more rapidly with several aspects of Chance and Data than others. Teaching and learning programs also vary and may, at times, inadvertently or deliberately emphasise some aspects of Chance and Data more than others.

Nevertheless, while the outcomes for Chance and Data are dealt with separately in these materials, they should be developing together and supporting each other, leading to an integrated set of concepts within students' heads.

The Markers of Progress for each mathematics outcome indicate the typical things students are expected to do at the same time. Generally, students who have access to a curriculum that deals appropriately and thoroughly with each of the outcomes reach a particular level at roughly the same time for each outcome in Chance and Data.

> *A student has demonstrated a marker of progress towards a **particular outcome** when he or she is able to do all the things described consistently and autonomously over the range of common contexts or experiences.*
>
> *A student has achieved a **set of outcomes** when he or she consistently and autonomously produces work of the standard described.*

Judgment will be needed to decide whether a student has achieved a particular level. When mapping and reporting a student's long-term progress, a teacher has to find the specific outcome level or the level for the set of outcomes that best fits the student, in the knowledge that no description is likely to fit perfectly.

The Chance and Data Markers of Progress are elaborated upon on pages 249 to 261.

CHAPTER 3

# Understand Chance

This chapter will support teachers in developing teaching and learning programs that relate to this outcome:

***Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments and analysis.***

## *Overall Description*

Students recognise that many situations are somewhat unpredictable, e.g. winning the netball, whether it will rain on the way home from school, or getting a good hand in a game of cards. They make appropriate use of the everyday language of chance such as 'might', 'could', 'likely' and 'unlikely', 'certain' and 'uncertain', 'possible' and 'impossible', 'probably', 'odds', 'fifty-fifty'. They realise that situations with uncertain individual outcomes may show long-term patterns in their behaviour, and that we use this to help interpret data and make predictions in order to address questions such as: 'How many mice will we have by next month?' 'What will the weather be like for our celebration in July?' 'How long will the battery last?'

Students compare events, using numerical and other information to order them from those least likely to happen, to those most likely to happen. They know that probability is the way we quantify how likely it is that something will happen, and they can interpret the probability scale from 0 to 1. They estimate probabilities from experiments and simulations, using the long-run relative frequency. They also use systematic lists, tables and tree diagrams to help them analyse and explain possible outcomes of simple experiments, and to calculate probabilities by analysis of equally likely events.

| Markers of Progress | Pointers<br>Progress will be evident when students: |
|---|---|
| Students show some recognition of the element of chance in familiar daily activities. | • respond appropriately to everyday language associated with uncertainty, e.g. 'will, won't, might', 'could, couldn't'<br>• talk about events in ways that show they recognise their chance nature, e.g. say *Our new baby might be a girl or might be a boy*<br>• use language such as 'won't happen', 'will happen' or 'might happen' appropriately, e.g. *Tomorrow it might rain* or *I need seven to win and that can't happen*<br>• recognise and take into account the possibility of different results for repetitions of the same simple action, such as throwing a die for a game, e.g. say *I got 3 last time but I won't always* |
| Students distinguish possible from impossible events and describe familiar, easily understood events as being more likely or less likely to happen. | • respond appropriately to, and use, 'possible' and 'impossible' for describing familiar events and actions<br>• identify possible and impossible results of a familiar simple action by thinking about the situation, e.g. *You might get a red or a green or a pink jelly bean because they are the colours we put in the bag, but you couldn't get a black one because we didn't put any in*<br>• identify possible results of an action or event by collecting data, e.g. *I tried it out and found the hoop could go over no pegs, one peg or two pegs, but it couldn't fit over three because they are too far apart*<br>• identify possible outcomes for daily events, e.g. *After Dad picks me up from school, we go to the shops, go straight home or go and visit a friend*<br>• distinguish impossible from unlikely events, e.g. *We never go to the park after school but it isn't impossible—it could happen*<br>• describe familiar events as being more or less likely to happen, e.g. *After school we are more likely to go …* |
| Students distinguish certain from uncertain things and describe familiar, easily understood events as having equal chances of happening or being more or less likely. | • use data to compare events within their experience, describing them as being more and less likely, e.g. *It is more likely to rain in Bunbury in July than in January*<br>• describe outcomes as having an equal chance or being equally likely, e.g. *a head has an equal chance with a tail, a baby is equally likely to be born on any day of the week*<br>• order a few easily understood situations from the least likely to most likely, e.g. *Tomorrow will be Sunday and my teacher will come to school tomorrow with green hair*<br>• justify their choice of more or less likely by referring to past experience or known information, e.g. say *I don't think my teacher will come to school tomorrow with green hair—it's almost impossible, but it's even less likely tomorrow will be Sunday because today is*<br>• make informal statements about how one might influence the chance of an event happening, e.g. say *I am less likely to have an accident if I take the back road because I don't have to cross any busy streets* |
| Students place events in order from those least likely to happen, on the basis of numerical and other information about the events. | • use available data to order things from least likely to most likely, e.g. using rainfall data to order capital cities from least likely to have rain in January, to the most likely<br>• order outcomes for a single random action from least to most likely by thinking about (i.e. analysing) the situation, e.g. for a die with the faces 1, 1, 2, 2, 2, 3, state that a 2 is most likely, 1 is next and 3 is least likely<br>• order probability devices from the one most likely to the one least likely to produce an outcome, e.g. order three spinners with different proportions shaded yellow from the one most likely to the one least likely to produce a yellow result<br>• design a probability device such as a die, spinner or a bag of coloured beads to produce a specified order of probability, e.g. colour a spinner so it is most likely to stop on red, least likely to stop on green and has the same chance of stopping on yellow as blue<br>• place informal expressions of chance on a scale from 0 to 1, e.g. 'impossible', 'poor chance', 'even chance', 'good chance' and 'certain' |
| Students interpret and make numerical statements of probability based on lists of equally likely outcomes, and using fractions and percentages. | • understand that events which cannot happen are certain as having a probability of 0, events that will happen are certain as having a probability of 1, and events which may happen as having a probability between 0 and 1<br>• interpret expressions of probability in general usage such as *The probability of rain tomorrow is 30%* and *There's a 50-50 chance*<br>• list equally likely outcomes for a 'one-step' action in order to assign probabilities, e.g. each of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are equally as likely to appear as the last digit of a telephone number (but not as the first digit), so the probability that the last digit is 7 is 0.1<br>• use fractions to assign probabilities, e.g. 14 girls' and 16 boys' names are put in a bag; the probability of drawing a boy's name is 16 out of 30<br>• use the results of a simple experiment to predict the results in a repetition of it, e.g. use results of tossing a hockey stick 50 times to predict what would happen in a later experiment of 100 tosses<br>• design a device to fit specified probabilities, e.g. colour a spinner so the probability of it stopping on red is 0.5, on green is 0.1 and on blue is 0.4 |

## Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU). These Key Understandings underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

| Key Understanding | Stage of Primary Schooling—Major Emphasis | KU Description | Sample Learning Activities |
|---|---|---|---|
| **KU1** Some things we are sure will or will not happen and other things we are unsure about. | Beginning ✔✔✔<br>Middle ✔✔✔<br>Later ✔✔ | page 12 | Beginning, page 14<br>Middle, page 16<br>Later, page 18 |
| **KU2** There are special words and phrases we use to describe how likely we think things are to happen. | Beginning ✔✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 20 | Beginning, page 22<br>Middle, page 24<br>Later, page 26 |
| **KU3** We can compare and order things by whether they are more or less likely to happen. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 30 | Beginning, page 32<br>Middle, page 34<br>Later, page 37 |
| **KU4** We say things have an equal chance of happening when we think they will happen equally often in the long run. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 44 | Beginning, page 46<br>Middle, page 48<br>Later, page 50 |
| **KU5** We can use numbers to describe how likely something is to happen. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 52 | Beginning, page 54<br>Middle, page 55<br>Later, page 57 |
| **KU6** Sometimes we list and compare all the possible things that could happen to predict how likely something is to happen. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 60 | Beginning, page 62<br>Middle, page 63<br>Later, page 65 |
| **KU7** Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 68 | Beginning, page 70<br>Middle, page 72<br>Later, page 74 |

**Key**

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

## KEY UNDERSTANDING 1

### *Some things we are sure will or will not happen and other things we are unsure about.*

In developing this Key Understanding, the emphasis should be on students recognising the element of chance in familiar daily activities. It should be developed in conjunction with Key Understanding 2.

Students' daily experiences involve a considerable element of chance. On the one hand, their emotional and physical security depends upon their capacity to predict (and hence control) aspects of their world. On the other hand, many things are not predictable or are predictable only within certain bounds. It seems that from quite a young age, children begin to look for causal explanations, partly so that they *can* predict what will happen in the future. By the time they come to school, they will assign causes for events which may seem funny to an adult. An example is the preschooler who announced the arrival of a little sister, saying *I don't have a brother because Mummy won't let me.*

Considerable life experience is involved in distinguishing those things that are subject to chance variation from those that are not. Events that appear unpredictable may simply be those we do not know enough about, and those that we regard as quite predictable can surprise us.

The essence of this Key Understanding is not that students are always able to correctly classify actions or events by whether chance is involved. Rather it is that they understand that some things are affected by chance processes and others are not. Things that are affected by chance processes are those that we cannot be certain of, that is, they are 'uncertain'. Those that are not affected by chance processes include things that must happen *and* those things that cannot happen. That is, impossible events are just as predictable as things that must happen (see Key Understanding 2).

Students should investigate and discuss actions and events that are affected by chance processes. These should include familiar events that are not random, but have an element of unpredictability about them (e.g. whether Nanna will arrive before they have to leave for school, whether the ball will go through the hoop, or which tunnel their pet mouse will choose), as well as actions or events we think of as random, such as drawing raffle tickets, using spinners and throwing dice.

## Progressing Through Key Understanding 1

Initially students begin to show that they recognise an element of chance in many things that are a part of their lives, showing that they understand expressions such as 'will happen', 'won't happen' and 'might happen' by how they respond to and use them. They are beginning to show some understanding that repetitions of chance actions are likely to produce different results, e.g. when choosing lucky dips they understand that they may get something they like this time, but not next time.

As students continue to progress they can list things that might happen in relation to daily events or actions, such as what they will do on Saturday or the various ways a top might land. Next, they have a reasonable sense of the difference between certainty and uncertainty. As students progress further, they understand that the essential nature of chance processes means that things which are very unlikely are still possible and that things which are very likely may not happen.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔✔

### Guess What?

Help students to distinguish between situations they feel sure about, and those they are unsure about. For example: When playing 'Guess Who' or 'Guess What' games, let students see you put a toy car under the cover. Ask: Can you be sure what's under the cover? Could it be anything else? Ask them to close their eyes while you put something under the cover and ask them to guess what it is. Ask: Can you be sure what's under the cover this time? Draw out the difference between being certain and making a guess.

### Is It Possible?

Extend activities such as the 'Guess What?' game above by asking students to think about what could be under the cover. For example, place a pillow under a blanket and ask: Could there be an elephant under the cover? Why not? What could possibly be under there? Have students take turns to suggest what it could be.

### Certain or Uncertain?

Use everyday classroom events to talk about uncertainty, contrasting these with things we can be sure about. For example: Who's going to be the messenger next week? We know because it's written on the roster. Or: We can't be sure who is going to be the sports monitor because we haven't decided yet.

### Could, Will, Won't

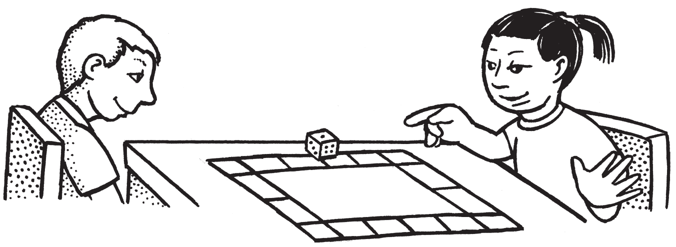
Give students activity cards showing different events such as riding a bike, watching television, and playing in the rain. Ask them to sort the cards into piles depending on whether the event could happen, will happen, or won't happen tomorrow. Ask: Why do you think that?

### Challenges

When students tell news, challenge their causal explanations, whether or not you think they are valid, to provoke them to explain. For example, a student may say *My new baby will be a boy because that's what Dad wants*. Ask: Does that mean it has to happen? Can Dad really decide whether the baby is going to be a boy or a girl? Has anyone's dad ever wanted a boy and not had one? Also challenge in cases where the explanation may be reasonable, e.g. *The doctor told Mummy and Daddy the baby will be a girl*.

### Board Games

Invite students to play board games that require a particular number to come up on a die. Afterwards, ask: Can you make sure that the number you want will come up? Does the position of the die before you throw it make a difference? Does wishing for your number help? Have students test each suggestion to see if it helps the number to appear. (See Key Understanding 7.)



### Tosses

Have students toss several red/blue counters and try to get all counters to land with one colour face up. Ask: Can you do anything to make sure that you get one colour every time you toss the counters? Why can't you be sure that you will get exactly what you want to come up? (See Key Understandings 4 and 6, 'Middle'; Collect and Organise Data, Key Understanding 3, 'Middle'; ***First Steps in Mathematics: Number***, Calculate, Key Understanding 2.)

### Who Will It Be?

When choosing students for tasks, place some names into a hat and ask: Which name do you think will be picked? Could we make it so that name does come out first?

### Random Events

Ask students to predict the outcome of random events, then test their predictions. For example, give students a box of Smarties™ and ask them to predict which colour they will get if they pick one without looking. Ask: What colour do you hope you will get? Could you get a white Smartie™? Why not? What colour could you get? Anne's favourite colour is red, so does this mean she will get a red one?

### What Colour?

After activities like the previous one, ask students to think about what might happen if the event is repeated. For example: Anne got a red Smartie™ that time; it's Steven's turn next, so will he get a red one, too? Could he get a different colour? Test their predictions and ask: Why didn't he get the same colour as Anne? (See Key Understanding 6.)

### Story Predictions

While reading a story to the students, ask them what might happen next and what cannot possibly happen next, e.g. after Goldilocks breaks Baby Bear's chair. Ask: Do you think the bears will come back? Do you think Goldilocks will eat Father Bear? Could the bed break? Could the bears talk?

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Who Will It Be?

Extend 'Who Will It Be?'(page 15) by asking students to think about the possibilities for the events that are certain. For example: We know it's not Kim, he did it this week and his name doesn't go in the hat; we know it's not Adriana because she's not in our class.

### Is It Possible?

Extend 'Guess What?' games (see page 14) by showing students a range of objects (e.g. pencil case, apple, ruler) that might be under the cover. Give them clues that allow them to eliminate certain possibilities, e.g. if the clue is 'It isn't edible', they can be certain that the object is not the apple. Draw out that while they can't be certain what is under the cover, there are some things it certainly isn't.

### Tricky Clues

Extend the previous activity by introducing clues that are more ambiguous, e.g. 'I have one of these at home'. Include items that may or may not fit this criterion. Ask, for example: Can you really be certain it isn't a skateboard? Is it possible I have one at home? Draw out the difference between unlikely and impossible events.

### Could, Will, Won't

Extend 'Could, Will, Won't' (page 14) by asking students to list other activities that could, will or won't happen after school today, and explain why. Draw out that while some students can be certain some things will happen, others can be equally as certain they won't happen.

### Are You Sure?

After the previous activity, ask students to consider the events they said 'will happen' and decide whether they can really be certain about them happening. For example: You said you will watch TV this afternoon, but could something happen to make that change? Can you really be certain?

### Possible or Impossible?

Ask students to use the words 'possible' and 'impossible' to explain their choices when classifying books as fiction or non-fiction. Draw out that non-fiction books are always about possible events, but fiction stories can include impossible events, which we can be certain will not happen.

### Hangman

Invite students to play Hangman. Give them the first letter of the word and ask them to guess each letter in order. Ask: If our first letter is 'g', which letters would it be possible to have next? Which would be impossible? How do you know? (See Collect and Organise Data, Key Understanding 5.)

### More Hangman

Extend the previous activity by allowing students to use a list of letter frequencies they have created from samples of text, or obtained from a website, to help them guess which letters are in the word. Ask: Does knowing which letters are used most often help you predict the letters in this word? Draw out that letter frequency helps us decide which letters are more likely, but it does not help us decide for sure.

### Will It Happen?

Ask students to decide whether or not they can be certain 'familiar' events will happen. Select things that are certain to happen (the sun will rise tomorrow), those that are certain not to happen (if I drop this glass, it will float in the air), as well as a range of uncertain events. Ask students to explain their decisions.

### Footy Tipping

Conduct a class footy tipping contest. After students have predicted which teams they think will win, compare predictions with actual results. Ask: Did the team you thought was most likely to win actually win? Why do you think this happened? Draw out that some things which may seem likely always have an element of chance affecting the outcome. (See Key Understandings 3 and 7; Summarise and Represent Data, Key Understanding 5.)

### Who's Next?

Invite students to predict who the next person will be to walk through the door. If they predict correctly, ask: How did you know that (person X) was going to come into our room? If they predict wrongly, ask: Why can't you be sure who will come into our room next? Draw out that some things happen by chance, and are not predictable.

### Weather Forecasting

Have students create a list of different weather types including fog, hail, snow, sunshine, cloud, drizzle, etc. Ask them to choose one as their weather prediction for the next day. After a couple of weeks of weather predictions, ask students to consider how predictable the weather was. Ask: Why were we sometimes wrong? Is it possible to predict the weather, or is it unpredictable? (See Key Understanding 3.)

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔

### Timetables

Ask students to write a timetable showing what they expect to happen in their next school day. Ask: Are you certain about any of these events? How can you be certain? Which events are you uncertain about? What do you need to know to be certain that they will happen?

### What Might Happen?

While reading a shared fiction story, ask students to predict what might happen next and to create a list of suggestions. Have students review the list to decide which events are possible and which are impossible in real life. They then go through their list of possible events and say which are likely and which are highly unlikely. Then they review the list of impossible events and say how they can be certain the events are impossible.

### Predict and Test

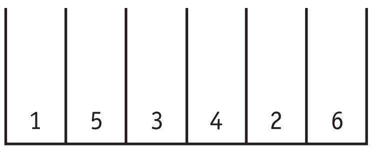
Ask students to predict and test the outcome of chance events after there has been a run of the same outcome. For example: Draw names out of a hat (replacing the name and shaking the hat each time) and have students list the names as they are pulled out. Before each name is pulled out, have them predict if it will be a boy or a girl. Ask: Can we be certain? What if we drew out four boys in a row? Would that change the likelihood of a boy's or a girl's name being drawn out next? (*No, because the number of boys' and girls' names in the hat hasn't changed.*) Under what circumstances could we be certain that the next name will be a girl? (*If the names were not replaced each time and all the boys' names had been pulled out.*) (See Key Understanding 5.)

### Luck of the Draw

Have students investigate the number of tickets that prize-winners have bought in a raffle. Ask them whether increasing the number of tickets someone buys increases their chances of winning. Ask: Is there any way you can be certain you will win?

### Possible or Impossible?

Ask students to decide whether events (including some that are very unlikely) are possible or impossible. For example, consider the 'laughing clowns' sideshow game. Ask: Using five balls, is it possible to get a score of 50? Of five? How do you know? Is it really impossible to get a score of five, or just very unlikely?



### Certain or Uncertain?

Invite students to classify the following situations as certain or uncertain:

- a porcelain plate will break if dropped
- you roll a die and get an 8
- you draw a green marble from a bag of red marbles
- you get either a head or a tail when you toss a coin
- it will rain tomorrow
- tomorrow I will be a day older
- tomorrow I will be a day younger
- our teacher will come to school with purple hair.

Ask: What factors do you have to think about when you make your decision? Will the plate always break or does it depend on the type of floor you drop it on? Ask students to re-sort the 'certain' situations into 'certainly will' and 'certainly won't' happen. (Link to Collect and Organise Data, Key Understanding 3.)

### Prize Box

Have students analyse various lucky dip situations, with and without replacing the selected object each time. In the situation where the object is not replaced, ask students to estimate how many times they would have to draw a chocolate bar to be certain they'd get, say, a Snickers™ bar (from a selection of five Cherry Ripes™, two Crunchies™ and one Snickers™). Ask: Why can't we say the same thing for the situation where we replace the bar each time? Draw out that it is possible to never get a Snickers™. (Link to Key Understandings 2 and 5.)

### Chance Cards (1)

Invite students to give examples of situations that match different likelihoods. For example, students take turns to select a chance card from a collection labelled 'certain', 'uncertain', 'very likely', 'likely' and 'very unlikely'. On a blank card, they write an example of a situation that matches their chance card. Ask them to share and justify the situations they wrote about. Draw out that very unlikely situations are still possible and very likely situations may not happen.

### Chance Cards (2)

Extend the previous activity. Ask students to exchange their situation cards with another group to sort in their own way. Have students compare their categories. Ask: Did you sort into certain/uncertain or possible/impossible? Encourage students to question the categories selected by others. Ask: Why did you choose certain, possible and impossible? Draw out that impossible situations are also certain. (Link to Collect and Organise Data, Key Understanding 3.)

## KEY UNDERSTANDING 2

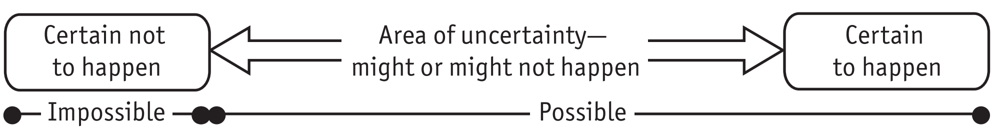
***There are special words and phrases we use to describe how likely we think things are to happen.***

The language of chance is widely used in a colloquial way, e.g. 'tomorrow will probably be fine', 'it's an odds-on bet', 'it's a sure thing'. In developing this Key Understanding, the emphasis should be on clarifying, refining and extending students' use of this everyday language of chance in conjunction with the development of Key Understandings 1 and 3.

Students' discussions of chance aspects of daily life and of their experiments should be based on natural language use, and ideas should be expressed in the students' own terms. Students need to hear terms such as 'possible', 'impossible', 'unlikely', 'likely', 'certain', 'probable' and 'improbable' modelled, and have opportunities to practise using them appropriately in context. From time to time, explicit attention should be paid to the use of expressions such as 'unlikely', 'it might happen', 'being lucky', 'that's not fair', 'always', and 'tomorrow it will probably rain'. For example, students in the middle and upper primary years might be asked to justify and explain their use of certain expressions based on past experience and the range of possible outcomes *(It's unlikely that we will go to Sam's house after school today because we usually only see him on the weekend)*, rather than on idiosyncratic associations based on single occurrences or feelings *(I bet it's going to rain today because I really want to do sport)*.

Students might try to place chance expressions in order from 'certain not to happen' to 'certain to happen'. (There is unlikely to be one right order; the intention is to consider the ways we use such expressions.)

The binary pairs 'possible-impossible' and 'certain-uncertain' are important, but they are different (see diagram, page 21) and will each require specific attention. Students who appear to distinguish things that must happen from things that might happen may have difficulty with the 'possible-impossible' distinction. Some may stretch the meaning of 'possible' to include almost anything and hence describe what adults regard as impossible events as 'might happen but don't!' Conversely, others may believe that things they have never personally experienced or do not wish to happen, cannot happen, and so stretch the meaning of 'impossible' to include things that are possible. Many students also find the notion that impossible events are 'certain' quite difficult. They should learn that, in English, we use the word 'certain' to describe events about which we believe there is no doubt, including those that must happen and those that cannot happen (e.g. *I am certain that I cannot get a black Smartie™ because they don't make them*).




## Progressing Through Key Understanding 2

Initially students show that they understand expressions such as 'will happen', 'won't happen' and 'might happen' by how they respond to and use them. As students continue to progress they are beginning to understand the idea of 'impossible' and to distinguish impossible things from those that are possible even if unlikely. They can tell you it is impossible to get a white Smartie™ from a pack, because there isn't any such thing. Next, they distinguish certain from uncertain things and know that certain events include those that must happen and those that cannot. As students progress further they can sensibly sequence chance-related expressions such as those listed above along a continuum. Later, they understand that probability is the way we measure chance, that is, probability statements give a numerical measure of how likely something is to happen.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔✔

### Language of Certainty

During conversations and news, respond to students' recounts, requests and descriptions of situations with comments such as: Did that really happen? I was not certain that you would be able to do that. Who else thought he could not do that? Use language such as: Are you certain you left it? If you are uncertain about how much you will need, go and check, then come back when you know, that is, when you are certain.

### Please Explain

During conversations and news, respond to students' use of certainty language by encouraging them to explain their meaning. For example: You said you will definitely go for a swim after school today. What did you mean?

### Likelihoods

Repeat the previous activity, but focus on language describing the likelihood of events. If a student says *I'm probably going to the zoo this weekend*, say: So it's quite likely you are going to the zoo. How do you know it is quite likely?

### Who Will Win?

Ask students to say who they think is going to win a running race between members of their class. Then ask how sure they are that their choice will win. Ask: Is it possible that Janie will win? Is it impossible for Mike to win? Refer to someone who is not running, and ask: Is it possible for Zeke to win?

### Picture Sort

Have students sort cards that depict related events, some of which are obviously impossible and others possible, e.g. an elephant sitting in a bus, a child sitting in a bus. Ask students to explain why they think the event is impossible or possible. (See Sample Lesson 1, page 28.)

### Wacky Wednesday

Read Dr Seuss's *Wacky Wednesday* (Lesieg, 1975) to students and ask them to identify the impossible events. Ask: How do you know the shoe on the wall was impossible? Why wasn't the hook on the wall impossible?

### What Will Happen? (1)

Provide students with cards showing pictures of classroom activities such as using a calculator, playing cards, reading a book, playing with play dough. Ask them to suggest words that describe the likelihood of each event. Ask: Will these things happen? Do we really know they will happen, or are we better to say they might happen? (See Key Understandings 3, 4 and 5.)

## What Will Happen? (2)

Repeat the previous activity for events that have been ordered. For example: You said you were more likely to do maths tomorrow at school than watch a video. Could we say we will probably do maths? Is it possible that we won't do maths?

## Lucky Dip

Invite students to wrap a number of items for a lucky dip game and then ask others to guess what might be in the packages. The students who made the game then use chance words to say whether each suggestion is 'possible' or 'impossible', 'certain', 'certain not to be' or 'maybe'. Record each example and then sort them to say what has/has not been used in making the game. Then ask the students playing the game to say what they can 'possibly' get in the lucky dip.

## Uncertainty Words

Collect 'uncertainty'-related words students use in their everyday speech, e.g. 'maybe', 'could be', 'might', 'perhaps'. Use the words to stimulate storytelling, drawing attention to the difference between events students have some control over and events that just happen.

## Is It Safe?

Ask students to help design a circuit of play equipment using tyres, boards, ladders, climbing frames and so on. Help them check the safety of their suggestions by asking: Is it more likely that children will fall off if the board is ...? How do you know? Where could the board go to make it less likely that children will fall? If we use Clare's suggestion and put the tyres in a zigzag, are more children likely to want to play on it? (Link to Key Understanding 3.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Correct Language

Ask students to consider whether everyday use of chance language is correct. For example: Sally said there is no way she could win the lottery—it's impossible. Do you agree with her? Can she be certain she will not win the lottery? How do you know? Draw out that events that haven't happened yet or haven't happened in their experience are not necessarily impossible.

### Possible Outcomes

During conversations and news, ask students to justify their use of chance language by referring to past experience or possible outcomes. For example: You said you were certain that the Dockers would win this week. What made you so sure? Ask: Is that the only thing that could happen? Can you really be certain? Model appropriate justifications that refer to possible outcomes or to past experience.

### Story Predictions

When reading stories, ask students to list events that could occur next. Encourage them to consider events that might happen, but don't happen often. Draw out that these events are still possible.

### Possible or Impossible?

Present students with things they will not expect to happen and discuss whether they are possible or impossible. For example: Sam said that it's impossible that an eight-year-old boy would drive a car. Was he right? Draw out that events might be possible even if you have never seen them happen or do not expect them to happen.

### It Could Happen

Ask students to list events that they are certain will happen, that might happen, and that they are certain won't happen. For example: Before an excursion, have students list various events and ask: How do we know these events will happen? Can we really be certain? At the end of the excursion, compare these lists with what actually happened. Were there some events that could have happened, but didn't?

### Weather Forecasting

Extend the 'Weather Forecasting' activity (page 17) so that students choose a weather type for the following day and then say how likely they think it is to happen. Record their responses and then consider why they all indicate a high likelihood of the event.

### Lucky Tickets

Have students investigate the number of tickets prize-winners have bought in a raffle. Ask: Does increasing the number of tickets you buy increase your chances of winning? Is there any way you can be certain you will win?

### Likely Events

Ask students to collect magazine cuttings and sort them into those depicting events that cannot happen (impossible) and those events that can happen (possible). Focus on their explanations and draw out the difference between impossible and highly unlikely. Ask: Can this ever happen anywhere? Could someone else ever see such a thing happening?

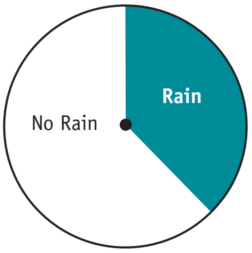
### Sporting Events

Have students review newspaper articles on sports such as football, and find words used to describe likelihood. Make a class collection of words and then place these in groups showing examples of words that seem to indicate the same likelihood.

### Science Certainties

When watching science documentary videos, e.g. on space exploration, ask students to make a list of the words used to indicate that some events are not certain. Ask: Why would these words be used?

### *Did You Know?*




We sometimes use the word 'random' as though it means haphazard, but randomness involves a kind of order that emerges in the long run. To a statistician, a phenomenon is called random if individual outcomes are uncertain but the long-term pattern of many individual outcomes is predictable.

We often deliberately plan randomness to produce a particular long-term pattern. For example, we may know from local meteorological records that it rains on one in three days in April. We could produce a spinner with a two-thirds sector shaded yellow to represent 'no rain that day' and a one-third sector shaded grey to represent 'rain that day'. We could use it to simulate a series of days and to study possible weather patterns for April in our location. The spinner represents the two possible unequal outcomes (rainy and not rainy). The result of any particular spin will be uncertain but, in the long term, one in three spins will result in grey representing a rainy day.

Another example involves dice that must be perfectly balanced. These are very carefully made with the pips representing the numbers on each face filled with the same material as the rest of the dice to ensure that the dice is balanced. In this case, the intention is to ensure that each of the six faces is equally likely to come up in the long run. Again, while the individual outcomes are unpredictable, the dice are carefully made to ensure predictability in the long-term pattern of outcomes.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Weather Check

Ask students to review television and newspaper weather forecasts or the Bureau of Meteorology website to find words used to describe likelihood. Ask: What do they mean when they say 'squalls possible in thunderstorms'? Can a weather forecaster be certain of the weather for the next day? How do they express their level of certainty?

### Word Sort

Have students cut out sentences from magazines and newspapers that contain chance words (e.g. nearly, may, Buckley's, possible, doubtful, highly probable, fifty-fifty, likely). They sort the sentences according to likelihood, e.g. unlikely, likely or certain. Ask students to say whether the certain events are those that will happen or those that won't. (See Key Understanding 5.)

### No Chance

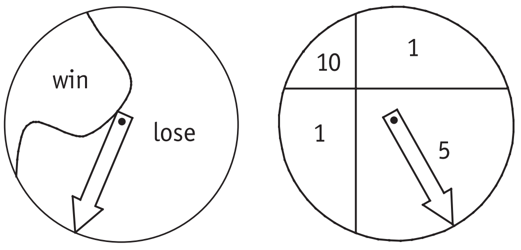
After activities such as the previous one, ask students to review their list of words to find those that suggest little or no chance. The words might include: never, seldom, no way, pigs might fly, unlikely, least likely, impossible, improbable, no chance, little chance, fat chance, slim chance, Buckley's, once in a blue moon, one in a million. Ask: Can you be certain that some of these events won't happen? Can we be certain an event is impossible?

### Influencing Events

Encourage students to use informal probability language to say how they can influence the likelihood of events occurring. Provide them with a list of events, e.g. having your wallet stolen, getting skin cancer, blue house/faction winning the sports, winning a raffle, getting to draw a prize from the class prize box. Ask them how they could make these events more or less likely to happen. Ask: Is it possible to influence things so that you can be certain they will or won't happen? (See Key Understanding 3.)

### Spinners

Ask students to use the language of probability to describe the likelihood of events happening. Students spin the two spinners and keep score as they go, according to whether they win or lose the different amounts shown. Ask: How likely are you to win/lose a large/small amount? (See Key Understanding 3.)



## Fitting Expressions

Have students suggest events they think fit some of the expressions below, and argue with their peers about why they think it so:

- a small chance
- quite certain
- more than likely
- a good chance
- almost certain
- not much chance
- highly improbable
- extremely likely
- a very good chance
- very likely
- fairly likely
- in all probability
- highly unlikely
- likely
- a 50-50 chance
- pigs might fly
- even chance
- Buckley's
- probably
- not in a month of Sundays.

## Chance of Rain

Invite students to use words and phrases such as those above to give meaning to numerical statements about chance. For example: There is a 10% chance of rain in the next week. Does this mean it is highly likely it will rain in the next week? Does it mean it certainly won't rain? What do you think it might mean? Extend to include other numerical statements. For example:

- the chance of getting a Snickers™ bar is 1 chance out of a possible 10
- the chance of one person drawing a heart from a hand of cards is 2 in 7
- the chance of getting a red Smartie™ is 3 out of 10.

Ask: What do these statements mean? Are they likely or unlikely, or is there an even chance that they might or might not happen?

# SAMPLE LESSON 1

**Sample Learning Activity:** Beginning—'Picture Sort', page 22

**Key Understanding 2:** There are special words and phrases we use to describe how likely we think things are to happen.

## Teachers' Purpose

I noticed that many of the students in my Year 1/2 class used words like 'impossible' and 'no chance' in their everyday talk, but I wondered what distinctions they made between possible and impossible events. Children often hear adults say things like 'I've got no chance of winning the lottery', even when they have a ticket. I realised that students may not often be exposed to the real meaning of impossibility as referring to things that cannot happen. I decided to provide an activity that could stimulate discussion about what is possible and what is not.

## Action and Reflection

*At a later time I planned to ask the students to re-sort the collection of possible events into those that must happen and those that might happen. They could then begin to understand 'certainty' in chance as a general concept that included all the things that can't happen as well as all the things that must happen. Knowing these ideas are difficult to grasp, I initially focused only on the 'can't happen/can happen' categories.*

The students and I spent some time cutting out pictures from magazines. I made sure the collection depicted a wide range of events, including some that seemed to me to be obviously impossible.

I suggested that we create a collage on the pin-up board under two headings: Possible (can happen) and Impossible (can't happen). I held up several pictures and asked them to think about whether what they saw in the picture could really happen in real life. *Is it really possible for a cow to jump over the moon like that, or is it impossible? Why do you think that?*

Pairs of students took piles of cuttings and sorted them into the two categories. They were to tell each other what they could see happening in each picture, then decide if it should go in the 'can't happen' pile, or in the 'can happen' pile.

Erin had an advertisement for milk that showed a drawing of a cow in a busy city street. She insisted it went in the 'can't happen' pile, while her partner Jason argued that it should go with the 'can happen' pictures. I asked Erin to explain why she thought it impossible.

*Because cows have to live on farms, not in the city. There's no grass to eat,* said Erin. Then I asked Jason to explain his reasoning. *It might have fallen off a truck or something*, he said. *A truck crashed and some cows ran off, I saw it on TV.*

Erin seemed to be focusing on what *should* happen, rather than what *could* happen, while Jason seemed to have a more conventional way of reasoning about what is possible.

## Drawing Out the Mathematical Idea

I asked a 'what if' question to draw out the distinction further. *What if it was a tiger?* Erin was sure that would also belong in the 'can't happen' pile, *because tigers live in a zoo and they can kill people*. Interestingly, Jason switched his decision to 'can't happen'. *I've only seen a tiger in a zoo, or it could be in a circus but they wouldn't let a tiger be there*, he explained, pointing at the picture. I realised Jason's judgments were based more on direct personal experience than on what might be possible, with the result that his reasoning seemed to be inconsistent.

The personalised ways Erin and Jason made their judgments were what I expected, but I decided to help them extend their ideas about impossibility. I asked Peter, who was sitting at the next desk listening to the conversation, what he thought. (Because of an earlier conversation with Peter I guessed he would be able to give a more conventional explanation.)

*No, it has to be in the 'can happen' pile. It's just standing there in the picture, he said, it's not driving a car or anything impossible. Cows and tigers could be standing like that anywhere, any animals could be*, explained Peter.

Erin was not really convinced; her ideas were still focused on what normally happens—cows and tigers just don't belong in the city. Jason, though, was ready to be swayed by Peter's explanation, and expanded on the idea. *And lions and elephants and pigs—they'd be 'can happen' as well. They can get there, maybe on a truck or something. It's only silly things in 'can't happen'—animals can't drive cars, they can't talk, they can't play cricket.* He had taken on a more general notion of impossibility.

## Challenging Current Ideas

I then asked another question to further challenge their understanding: *What about if it was a dinosaur just standing there, not doing anything silly?* Jason hesitated, looking at Peter for guidance. Peter, though, was quite sure about this situation as well: *Dinosaurs got 'stinct and they're only bones now so it's got to be a 'can't happen' thing*. Jason was happy to agree.

***I helped Jason notice and resolve an anomalous or conflicting idea without actually correcting him or simply telling him 'how to do it'.***

I asked Peter to tell the whole class his reasoning about Erin's and Jason's picture, and I went on to reinforce this more conventional way of thinking about impossibility. There was opportunity to revisit the language when the collage was in place, and we began to try to separate the 'possible' group into those things that 'might happen' and those things that 'must happen'.

## KEY UNDERSTANDING 3

***We can compare and order things by whether they are more or less likely to happen.***

Just as we can compare and order objects and spaces according to size, or events by how long they take, we can compare and order events by how likely they are to happen.

In developing this Key Understanding, students should be assisted to draw on their experience to describe familiar things as more or less likely. They could say that it is more likely they will get run over if they try to cross at the highway than if they take the underpass, or that where they live it is more likely to rain in December than June. Having compared two events in this way, they should be assisted to put several events in order from those they think least likely to those they think most likely. For example, they could use expressions such as 'very likely', 'quite likely', 'equally likely as not', 'quite unlikely', 'very unlikely' (see Key Understanding 2) to describe and order unrelated events such as:

- we will do some mathematics in school today
- the egg will crack if I drop it
- it will rain today
- my teacher will come to school with purple hair.

They could also order related everyday events such as the likelihood of four possible destinations after school (home, the shops, the pool, aunty's place), explaining their reasoning.

Just as we would help students first develop the idea of area by comparing regions of obviously different areas, so too the idea of 'how likely' will be best developed if initially the events being ordered are obviously different in likelihood. As suggested in Key Understanding 1, students' lack of experience may result in them suggesting orders that seem odd to an adult. The criteria for evaluating the order of events students produce should relate to whether their explanations show an understanding of the idea of 'more likely', rather than whether they understand the events themselves or have sufficient experience or knowledge to make an accurate assessment of likelihood.

As they gain experience, students should be asked to order events that are closer in likelihood and, during the upper primary years, begin to place events in order based on numerical or measurement information provided to them, or on frequency data collected from their experiments (see Key Understanding 7). They should also develop the understandings necessary to make probability devices such as dice, spinners or bags of coloured balls to produce specified *orders* of probability (e.g. make a spinner that is most likely to come up red and equally likely to come up blue or green).

## Progressing Through Key Understanding 3

Initially students understand that some things are more likely than others. For example, on a warm and cloudless summer's day they will say it is more likely that it won't rain than that it will.

As students continue to progress they draw on their personal experience to describe familiar things as more or less likely, and provide relevant reasons. For example, they may say that it is more likely they will go to the shopping centre than straight home because they go to the shops on the way home most days (non-relevant reasons might be that they like going to the shops).

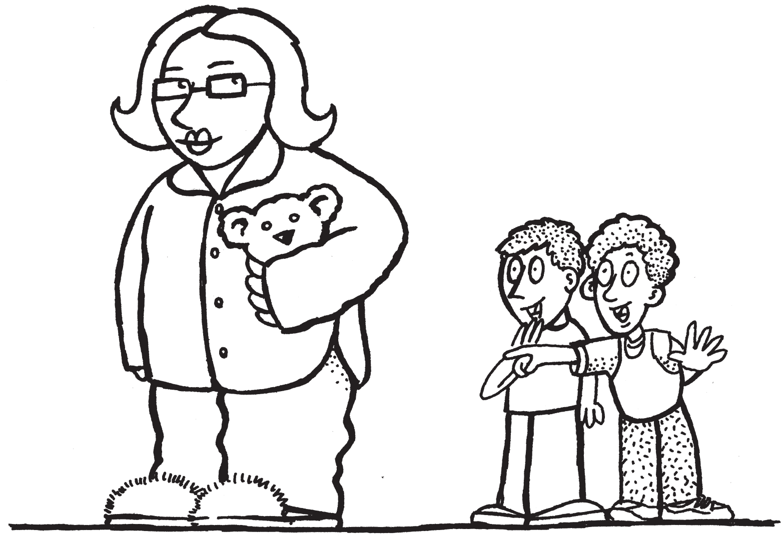
Next students are able to use a range of sources of information to put things in order from least likely to most likely. For example, they could use published data to order towns from the one most likely to have an earthquake to the one least likely. They could also count how many balls each group member was able to throw into a bucket in ten tries and use this data to order them from the one most likely to get a ball in on the next try to the one least likely to.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔

### More or Less Likely? (1)

Ask students which of two unrelated events is more likely. For example: Is it more likely that we will start school on time tomorrow or that the principal will come to school in her pyjamas? Choose events that are obviously more or less likely, since the intention is to focus on the meaning of the expression 'more likely'.



### More or Less Likely? (2)

Repeat the previous activity for two related events. For example: Are you more likely to go straight home after school today or to drive to Alice Springs? How do you know you won't go to Alice Springs?

### What Will Happen?

Provide students with cards showing pictures of classroom activities such as using a calculator, playing cards, reading a book, playing with play dough. Ask them to order the cards from most likely to least likely to happen at school tomorrow. Ask: What makes you think that we are very likely to read a book tomorrow? (See Sample Lesson 2, page 40.) (See Key Understandings 2, 4 and 5.)

### What's for Lunch?

Before lunchtime, ask students to predict what they are most likely to have for lunch. Record their predictions and the actual contents of their lunch for a week. Ask: How did you know what you were likely to get for lunch? What are you most likely to get tomorrow? (See Key Understanding 7; Link to Reason About Number Patterns, Key Understanding 1.)

### More Lunch

Following the previous activity, ask students to list what they are most likely and least likely to have for lunch for the next week, and the reasons for each decision. Ask: How often do you think you will get the items on your 'most likely' list? How often will you get the items on your 'least likely' list? Why do you think that?

### Story Predictions

When reading a story to students, stop periodically and ask them to predict what might happen next. Write each suggestion on a card and have students order the cards to say which is most likely and least likely to happen next. Ask: What is in the story that helps you to decide which is most/least likely?

### After School

Ask students to record their after-school activities for a week or so, and then predict what they are likely to be doing over the next few afternoons. Ask: What are you most likely to be doing? Why do you think it is likely? Why do you think (name an activity) is more likely than (name another activity)? (Link to Reason About Number Patterns, Key Understanding 1.)

### Two Steps Forwards

Have students toss an uneven object, e.g. drawing pin, bottle top, button, drink umbrella. When it lands face up, the thrower takes two steps forwards and when it lands face down, they take two steps backwards. Have students play with a partner and see who can be first to a given line. Ask: Is the drawing pin likely to land face up or face down? Which side is more likely? Would it be better to take forward steps when it lands face down instead? (See Key Understandings 4, 'Beginning', and 7, 'Middle'.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Will It Rain?

Ask students whether they think it is likely to rain today/tomorrow. Ask: What helps you make your prediction? (Talk about seasons and weather predictions.) Have students record their predictions and compare with the actual weather. (Link to Key Understanding 1 and Reason About Number Patterns, Key Understanding 1.)

### Footy Tipping

Hold a class footy tipping competition, with students predicting which team will win each week. Ask: How did you decide which team is more likely to win? Discuss which predictions were based on evidence and which were based on such things as loyalty. Ask students which strategy is likely to give the best predictions. (See Key Understandings 1 and 7; and Summarise and Represent Data, Key Understanding 5.)

### How Can It Change?

Have students brainstorm factors that might change the probability of an event happening. For example: What could you do to make it less likely that you will have an accident in the playground, or will be late for school?

### Dice Games

| | | | | |
|---|---|---|---|---|
| | | 8 | | 3 |
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| Answer | | | | |

Have students toss a ten-sided die and put the digits into a grid (see left) to build a three-digit number and then a four-digit number that will add to the largest amount. Ask: If the first number is 3, where would you put it? If the next number is 8, where would you put it? Would you be likely or unlikely to get a number larger than 8? For each toss of the die, consider the likelihood of other numbers coming up that are larger or smaller.

### Wheel of Fortune

Play a 'Wheel of Fortune'-type word-guessing game on the whiteboard. Allow students to choose, say, three consonants and one vowel. Place these in the appropriate slots and ask children to guess the word. Ask: How did you know which letters to choose? What would happen if you pulled letters out of a hat instead of choosing them? Draw out the fact that some letters are more common and thus more likely to be needed. (Link to 'Hangman', page 17.)

My consonants are D, S and T.
My vowel is E.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | d | | | | | | s | | | d |

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| t | | e | t | | | e | e | | e | | | s |

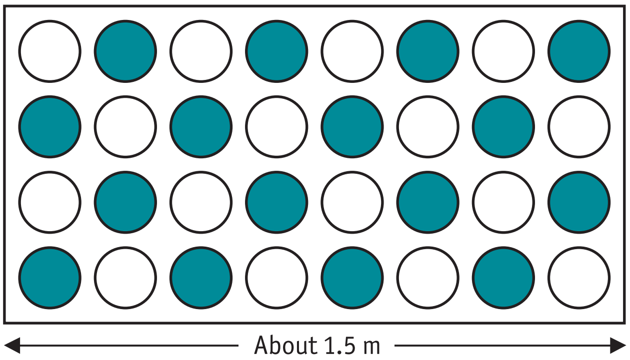
Answer: Goldilocks and the Three Bears.

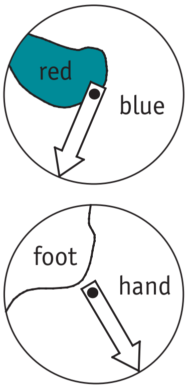
## Weather Forecasting

Ask students to list different types of weather conditions, then place them in order from the least to the most likely to occur tomorrow. Ask them to justify their decisions. (See Key Understandings 1 and 3.)

## Round the Twist

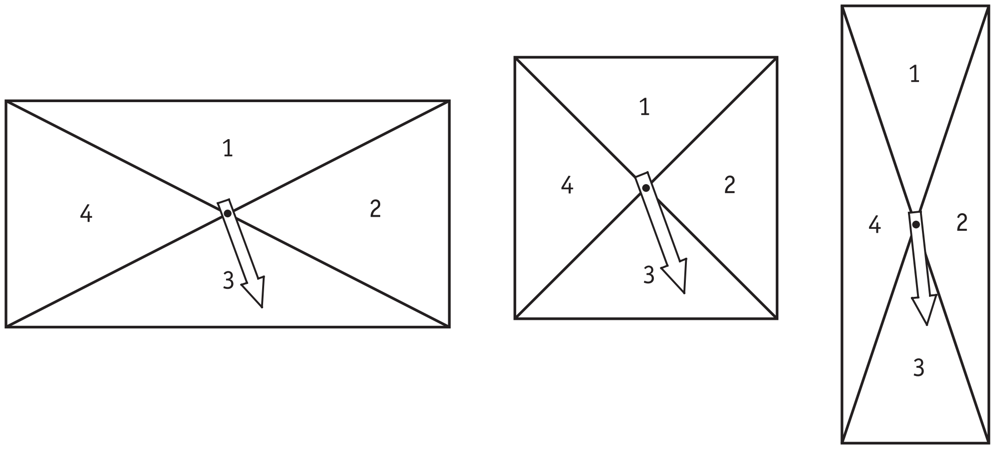
Construct a modified version of Twister™ using a 4 x 8 array of red and blue circles. Provide students with two spinners such as those below. To play, the teacher spins both spinners and a student follows the instruction, that is, puts a foot or hand on a red or blue spot. This is repeated for several more students, then turns are rotated through the players, each trying to follow the instructions on the spinners without falling or knocking other players over. After the game, ask: Are you more likely to get red or blue? Are you more likely to get a hand or a foot? Why?




## Making Spinners

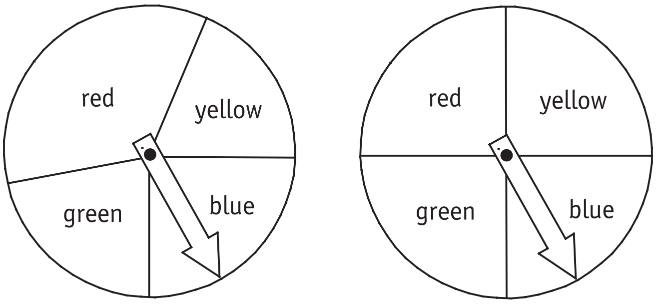
Invite students to make their own spinners and use them for games such as Snakes and Ladders. For example: Rectangular spinners can be made using a lid from a shoebox by marking the centre point of the rectangle and connecting this with the corners (see below). Have students compare a spinner with unequal sections with one that has equal sections and say why some numbers are more likely to come up than others. Ask: If you wanted to get a 4, which spinner would you like to use?



# Middle ✔✔✔

## Comparing Spinners

Show students a range of spinners, some with four equal sectors, others with unequal sectors. Ask: If you need red to win a game, which spinner would you choose? Why?



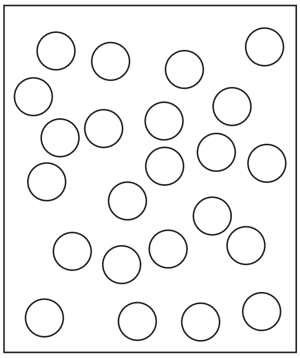

## Pattern Block Pictures

Ask students to make a picture using Pattern Blocks and a six-sided die that has its faces marked with three triangles, two hexagons and one rhombus. They roll the die a set number of times, collecting the shapes that appear. They then create their picture or design with those shapes. Ask: What shapes are impossible to get? Which shape are you most likely to get? Why can't you be certain you will have a hexagon in your picture?

## Smarties™

Give students diagrams of a number of scattered circles to represent a collection of Smarties™. Say: Pretend the Smarties™ will be put in a bag and you will be selecting one without being able to see which one you are selecting. Colour them in to produce these results:

- Colour the Smarties™ so you would be certain to get red. (Link to Key Understanding 2.)
- Colour the Smarties™ so that it would be impossible to get green. (Link to Key Understanding 2.)
- Colour the Smarties™ so you would be more likely to get red than green, and more likely to get green than blue. (Link to Key Understanding 3.)
- Colour the Smarties™ so you would be equally likely to get red, green or blue, but could not get any other colour. (Link to Key Understanding 4.)

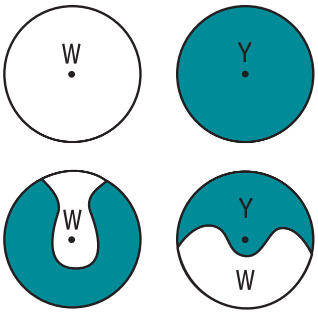


# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

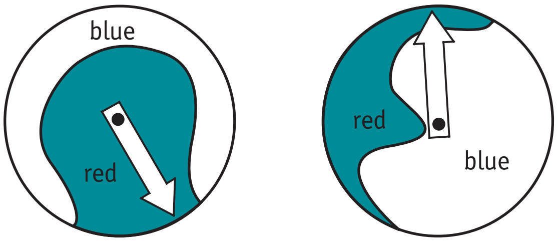
### Ordering Spinners

Have students order spinners (see below) according to how likely they are to result in particular outcomes. For example: You need to spin white to win. Put the spinners in order according to which one you would rather have. Ask: Why did you put them in that order? If you needed yellow to win, how would the order change? Why?



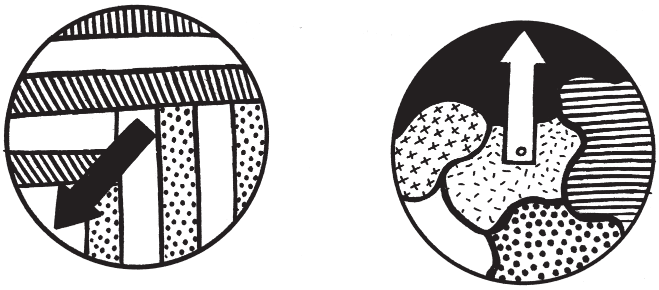

### Which One?

Present students with spinners such as the ones below. Ask: If you had to choose a colour to spin before you could start a game, which colour would you choose from the first spinner? Which would you choose from the other spinner? Why?




### Wrapping Paper Spinners

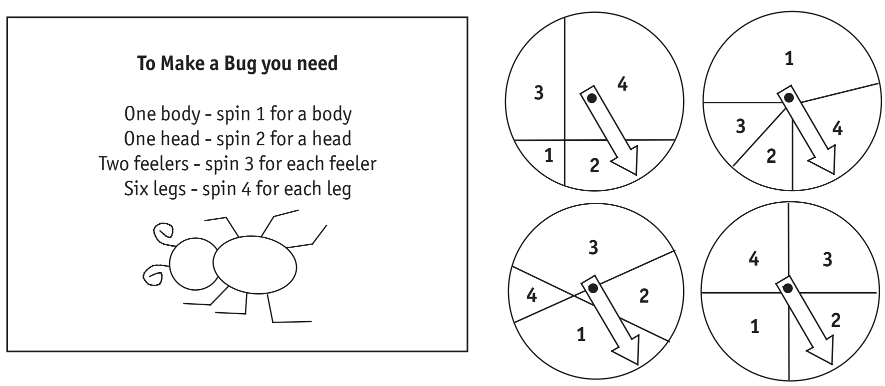
Ask students to use patterned wrapping paper to make spinners. Without looking at the pattern, they cut out an indiscriminate circle and place a spinner in the centre. Ask: Which colour is most likely/least likely to come up?



# Later ✔✔✔

## Different Spinners

Present students with a variety of spinners designed to land on numbers from 1 to 4. Ask students to choose one of the spinners to use in a game such as 'Make a Bug' (see below). To start the game they need to throw a 1 for the body and then roll a 2 to draw a head, 3 for a feeler and 4 for a leg. Ask: Which spinner would you choose if you want to draw a head? Which would give you the best chance of being able to draw a feeler? Which spinner would you choose if you could only use one throughout the game? Would you choose a different spinner if you could draw the parts in any order? Why?




## Cards

Ask students to compare the likelihood of drawing particular cards in different situations. For example: When selecting one card from a hand of five spades, three hearts and one club, which suit is most likely to be drawn next? Which suit is least likely to be drawn next? Why do you think that?

## Influencing Events

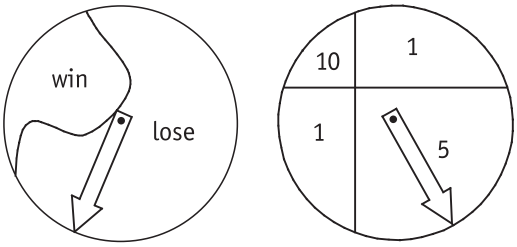
Invite students to discuss how they can influence the likelihood of events occurring. Provide them with a list of events, e.g. having your wallet stolen, getting skin cancer, blue house/faction winning the sports, winning a raffle, getting to draw a prize from the class prize box. Ask students how they could change the circumstances surrounding these events to make them more likely and then less likely to happen. (See Key Understanding 2.)

## Make a Spinner

Have students make probability devices, such as dice, spinners or bags of coloured balls, so that some events will have more chance of occurring than others. For example: Make and test a spinner that you think is most likely to stop on red, least likely to stop on green and have the same chance of stopping on yellow and blue. Ask students to compare the different ways they constructed their spinner. (See Key Understandings 4 and 6.)

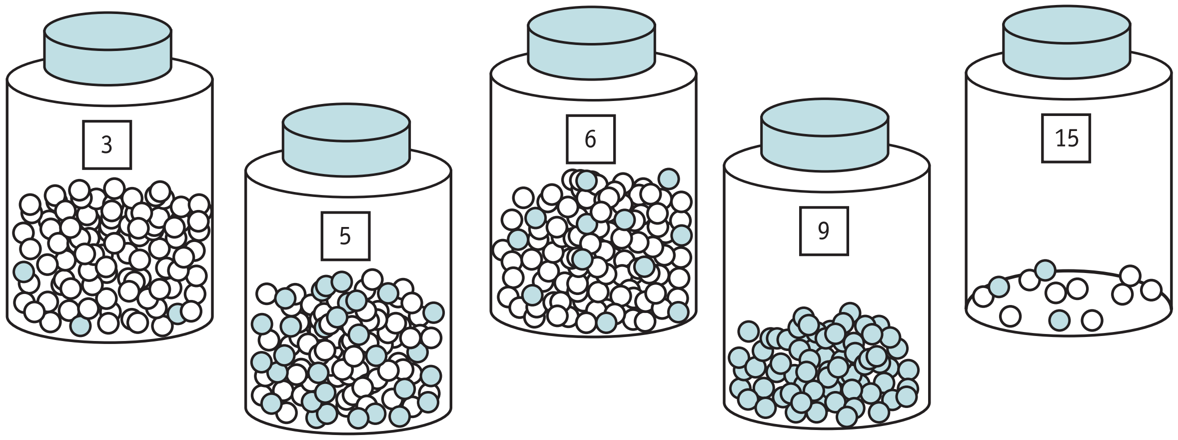
## Spinners

Ask students to examine the spinners below and decide what they think the outcome will be. Are you more likely to win an amount or lose an amount? Are you more likely to spin a 10, a 5 or a 1, or are they equally likely to come up? Explain your thinking. (See Key Understanding 2.)




## Chance Situations

Invite students to examine and compare different chance situations and predict which one is more likely to generate a specific event. For example: Provide a range of large, sealed plastic jars that contain different quantities and proportions of red and white beads. Have students work with two jars at a time, handling them and turning them around to examine the beads inside. Ask: If you put your hand into a jar and pulled out a bead without looking, from which jar would you be most likely to draw a red bead? How did you decide? What makes it difficult to choose? Have them swap jars around and repeat the process. (See Sample Lesson 3, page 42.)




## Pigs Might Fly

Ask students to order the likelihood of unlikely events in their daily life or from a book. For example: List some of the unlikely events that occur in the book *Pigs Might Fly* (Rodda, 1996). Rate these from Force 1 to Force 10, as is done in the book.

# SAMPLE LESSON 2

**Sample Learning Activity:** Beginning—'What Will Happen?', page 32

**Key Understanding 3:** We can compare and order things by whether they are more or less likely to happen.

## Teaching Purpose

When listening to my Year 3 students talking to each other I noticed that many referred to more and less likely everyday events using appropriate language. For example, I heard Kenny say to Gavin, *Yep, I'll probably be allowed to play at your house today*. And when Gavin asked if he'd also be allowed to sleep over, Kenny said, *Not much chance of that, it's school tomorrow*. I decided to give them an activity that would help them think about how we can directly compare and order a range of events according to how likely they are to happen.

## Action and Reflection

I thought of ten different but familiar events, most of which obviously varied in likelihood, and recorded them on cards. Some were about everyone in the class while others were personal and likely to cause disagreement. To give students a clear focus, I asked them to think about how likely these events were to happen tomorrow, rather than in general. The events were:

- We'll drink some water.
- We'll do some reading in class.
- A dog will bark.
- Someone in our class will use the toilet.
- We'll all stay home from school.
- One of us will travel on a plane.
- I'll ride on a bus.
- My teacher will wake up on the moon.
- It will rain.
- I won't watch TV.

*Students find it difficult to work out what we're talking about when we use terms like 'on average', 'normally' and 'in general' to refer to events. For example, asking a young child to think about the likelihood of going shopping after school* in general *requires a more complex set of judgments than asking about the likelihood of going shopping after school* tomorrow.

I divided the class into four groups and gave each a full set of cards. First they worked in pairs, each pair taking two of the cards and deciding whether one was more likely to happen than the other, and why. They then joined with another pair in their group and tried to order the four cards. Finally each group together ordered all ten from least to most likely.

Most in Abbie's group thought drinking water and going to the toilet had to go together, saying *We all have to do both things every day so one can't be more likely*. Abbie disagreed, saying that going to the toilet had to be more likely. I tried to get her to justify this assertion by asking: *Why do you think it is more likely?*

She replied: *We could drink cordial or cool drink, we don't have to drink water, but everyone has to go to the toilet*. This was surprisingly advanced thinking for this level.

## Drawing Out the Mathematical Idea

Kathryn and Ellie were arguing about whether riding on a bus was more likely than rain.

*So tell me why you think riding on a bus is more likely, Kathryn*, I said.

*Well, I go to after-school care every day on the bus, and Mum and I go to the movies at the city most Saturdays, and it doesn't rain as often as that,* she said. *What about you, Ellie? What do you think?* I asked.

*I went on a bus once last Easter when we went to Grandma's, and it's rained lots since then,* claimed Ellie, *so rain's got to be more.*

*So what is the same and what is different here?* I asked. *You ride on the bus a lot, Kathryn; Ellie hardly ever rides on the bus. Ellie said it's rained a lot since Easter; Kathryn said it doesn't rain very often.*

*But the rain can't be different,* said Ellie. *The rain's got to be the same for us both. If it rains, it rains. We both have to have the same chance of raining.*

*I get it,* said Kathryn. *The rain's the same but riding on buses is different.*

*So what can you say about the chances of these things happening tomorrow?* I asked.

*It can be both things,* said Kathryn. *I've got more chance of going on a bus, because Ellie's Mum picks her up in a car. It's more chance of rain for Ellie, because she hardly ever goes on a bus.*

I invited Kathryn and Ellie to explain what they'd found out to the class.

# SAMPLE LESSON 3

**Sample Learning Activity:** Later—'Chance Situations', page 39

**Key Understanding 3:** We can compare and order things by whether they are more or less likely to happen.

*I purposely chose to partly fill large plastic jars so that children could more easily examine their contents and see the relative proportions of red and brown beads, without opening the jars. The 16 jars were made up as follows:*

**Jars 1 to 7: half full of beads**

*Jar 1*—all white
*Jar 2*—all red
*Jar 3*—white with 3 red
*Jar 4*—red with 3 white
*Jar 5*—half red and half white
*Jar 6*—one-quarter red and three-quarters white
*Jar 7*—one-quarter white and three-quarters red

**Jars 8 to 14: quarter full of beads**

*Jar 8*: all white
*Jar 9*: all red
*Jar 10*: white with 3 red
*Jar 11*: red with 3 white
*Jar 12*: half red and half white
*Jar 13*: one-quarter red and three-quarters white
*Jar 14*: one-quarter white and three-quarters red

**Jars 15 and 16: 12 beads each**

*Jar 15*: 9 white and 3 red
*Jar 16*: 9 red and 3 white

## Teaching Purpose

My Year 6 students had had many experiences using informal reasoning to order everyday chance events according to how likely they are to occur. I decided they were ready for a more structured experience that required them to think about number when making their judgments.

## Action

I made up 16 large plastic jars containing varying numbers of red and white wooden beads. In small groups they compared pairs of jars, deciding which gave the best chance of drawing a red bead. We had a whole-class discussion, with pairs telling the rest of the class how they made their judgments. This gave students opportunity to use language such as 'very likely', 'fat chance', 'impossible', '100%', 'more chance' and 'less chance'.

After they had compared a few pairs of jars, I asked them to make a poster to record their judgments for any jars they found particularly interesting or tricky to compare. This gave me opportunity to move around and interact with the groups—manipulating some of the comparisons to draw out particular ideas. For example, Roselyn's group were comparing Jar 3 (half full of beads) and Jar 10 (quarter full of beads). Each contained three red beads with the rest white. By turning and shaking the jars they decided there were just three red in each. *So, that means they have to have the same chance*, said Roselyn. Others in the group agreed.

## Challenging Current Ideas

I did not question Roselyn's judgment, but arranged for the group to swap Jar 10 with another group's Jar 15 (9 white, 3 red beads), which they then compared with Jar 3 (half full of white with three red). Initially Roselyn said, *Oh, it's three again so that's just the same*, and others started to agree.

I asked: *Are you sure it's the same chance of getting a red from these two? If the reds were sweets and you had one chance to put your hand in and take one ball without looking, which jar would you choose?*

Ivan immediately grabbed the jar with the smaller number of beads. *I choose this one*, he said. *You've got much more chance of getting a sweet*.

*But why?* I asked. *There's only three red in each, so how can it make a difference which jar you draw it from?*

*But you can see, just by looking at it,* said Ivan.

*But what are you looking at,* I persisted, *what makes it different?*

Roselyn, who previously only focused on the number of red beads, recognised that the quantity of white beads was also important. *You have to look at how many white ones too—not just red ones. You'd be more likely to get a white in that one, even if the red's the same chance.*

I then retrieved Jar 10 and asked them to re-think their previous decision. Roselyn immediately questioned her earlier judgment. *I think it can't be the same, look, there's more white ones in that one.*

Ivan wasn't so sure. *But there's kind of a lot of white anyway. I think there's too many whites to make a difference. It'd still be hard to get a red.*

*Primary students will not be expected to understand how we can use numbers to describe proportional relationships. Nevertheless, they may use an intuitive sense of proportion to compare situations such as these involving mixtures in different ratios.*

*I was not concerned that there were still inconsistencies in their thinking. I knew these were very complex ideas and they would need more time and many more such experiences to develop these understandings.*

# KEY UNDERSTANDING 4

***We say things have an equal chance of happening when we think they will happen equally often in the long run.***

The analysis of 'equally likely events' is generally the basis for the calculation of probabilities (as developed in Key Understanding 6), and so it is important that students develop sound foundational ideas about what we mean when we say that events are equally likely. In the same way that two quite different looking regions could have the same area, two entirely different events could have the same chance of happening. Students should develop this understanding both through reasoning and practical experiences.

Being able to say that two events are equally likely does not require students to have any idea what the numerical probability is. Focusing too early on numerical probability is likely to be unhelpful and obscure, rather than helping the development of this concept. Students can be helped to understand what it means to say that two events are equally likely, by thinking about events that are more and less likely. For example, if given a list of events to order which includes both 'a head will come up on the toss of a coin' and 'a tail will come up on the toss of a coin', students will be confronted with deciding which is more likely. In the absence of any convincing reason to believe one or the other is more likely, the notion of being 'equally likely' should arise. Similarly, you might present students with a cube where each of the faces is a different colour, and ask them to try to put the colours in order from the most to least likely to appear. This will confront them with having to choose, with no reason to believe one is more likely than another.

Students should experiment with situations about which they have made predictions of equal likelihood. Such 'experiments' require careful handling, however, since it is in the nature of chance events that the outcomes are unpredictable! The difficulty for students beginning to make sense of chance is that equal chance does not mean that we expect the same empirical results in small trials.

However, students may expect 'equally likely' outcomes to appear equally frequently, even with small numbers of spins or tosses, and may lose confidence when they do not. This expectation may also lead them to expect the numbers to 'even out', as if chance is a self-correcting mechanism that promptly takes care to restore the balance whenever it is disrupted. Typically, students do not understand that each toss of the die is independent of what happens before or after. They need to systematically record the outcome of large numbers of dice rolls, for example, to be convinced that each number will come up equally often *in the long run*. They also need to compare the variability in small numbers of tosses with the relative predictability of large numbers of tosses.

The idea of equally likely outcomes is also closely related to notions of fairness. Thus, because we expect and rely on each face on a coin or die 'coming up' equally often, we would say that a coin or die is 'unfair' or biased, if the faces were unevenly weighted so that in the long run they would not come up equally often.

Students should investigate random devices for fairness. In some cases, deciding whether two events are equally likely will involve ideas about ratio. For example, recognising there is an equal chance of getting a red ball from two jars, one containing 20 red and 40 blue balls and one containing 40 red and 80 blue, requires a basic understanding of proportion.

## Progressing Through Key Understanding 4

Initially students may not distinguish situations that involve equally likely events from those that do not. As students continue to progress they do understand what it means for simple events to be 'equally likely'. They will say that a spinner coloured in four equal sectors is equally likely to stop on any of the colours, but that this is not true for another spinner coloured in four unequal sectors.

Next, students will use provided numerical information such as the average number of rainy days each month, or the number of each colour in a jar of balls, to decide whether two simple events are or are not equally likely to occur. As students progress further they can use their understanding of equivalent fractions to judge equally likely events.

# SAMPLE LEARNING ACTIVITIES

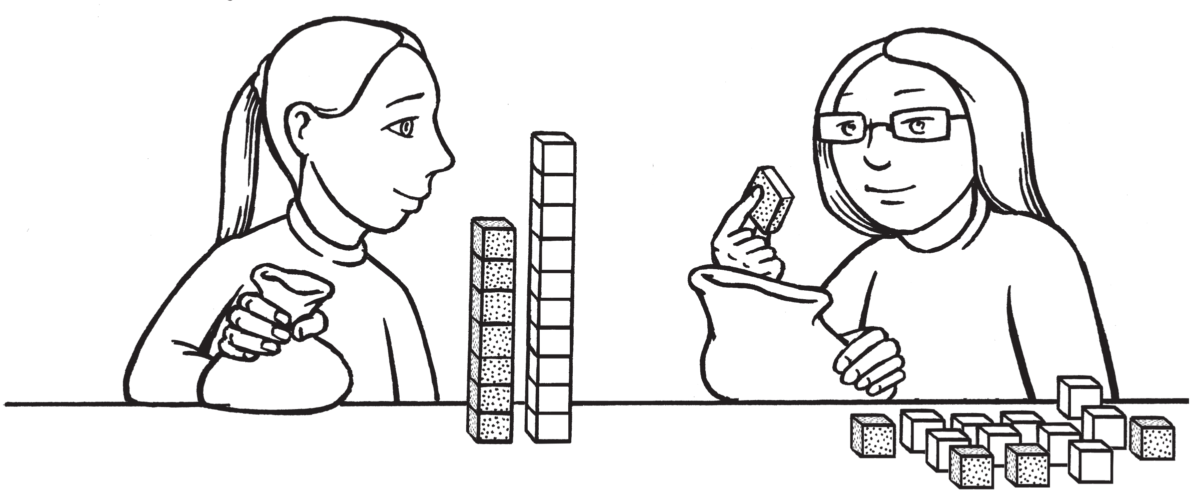
## Beginning ✔✔

### Fruit Time

Prepare fruit to be shared out onto plates. Ask students to predict what might be on the plates. Ask: Will we have more pieces of apple or pear? Why? (Responses might include *There were more apples than pears in the fruit box* and *We always have more apples than pears*.) Will there be more apples on this plate than that? *(They should be the same because we try to share them fairly.)*

### Towers

Ask pairs of students to cooperate to build, e.g. a blue tower and a yellow tower. Give each pair of students a feely bag that contains two square tiles (one yellow and one blue) and a pile of yellow and blue blocks. They take turns to draw out one square and add a block of that colour to the correct tower. The square is replaced after each turn. Before the game begins, ask students to say which tower will be taller. Focus on the suggestions that they could be the same and that either could be higher. As the towers grow, interrupt the game and ask the students to observe all of the towers built and say why they are different.



### Same Chance

Invite students to use playing cards to play Snap. When they have finished, ask how they should play so that each player has the same chance of being able to 'snap' the cards. Ask: What do we need to think about to make it fair? Does it matter if you look at a card before you put it down? Does where you sit and how you sit make a difference? What rules could we make to give everyone the same chance of saying 'snap'?

### Fair Choice

Ask students to decide whether random methods used by the teacher for assigning jobs or picking groups (e.g. picking names out of a hat) are fair. Ask: Could anyone in the class be picked? Does everyone have the same chance? Can we make it fairer?

### What Will Happen?

Provide students with cards showing various classroom activities such as using a calculator, playing cards, reading a book and playing with play dough. Ask them to place the cards in a line to show which are most likely to happen at school tomorrow to least likely to happen. Ask: Were there any you had trouble deciding between? Why were these two so difficult to order? (See Key Understandings 2, 3 and 5.)

### Two Steps Forwards

After playing 'Two Steps Forwards' (page 33), have students play it again using a red/blue counter. After the game, ask whether they prefer to play it with a counter or a drawing pin or drink umbrella. Ask: Would you expect to have red and blue come up equally often? Why is it better to play the game with something that does not have an equal chance of coming up?

### Building Spinners

Ask students to construct their own spinners and use them for games such as Snakes and Ladders. Invite them to compare a spinner with unequal sections with one that has equal sections and say why some numbers do not have an equal chance of coming up. Ask: Should we try to make all the sections the same?

### Jam Sandwich

Read *The Giant Jam Sandwich* (Lord, 1988) to the students. Then ask them whether they think a slice of buttered bread that is dropped is likely to land with its buttered side up, or whether either side has an equal chance of facing upwards. Have students test out their predictions individually and then combine their results to get a large sample. Ask: Which side landed up the most? Do you think there was a reason for this happening? If you did the testing again, would you get the same results? Why or why not?

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Tosses

Have students toss counters that have a different colour on each side (e.g. red/blue counters) and score points depending on whether the counters land with, e.g. red or blue facing up. Before they begin, ask them to predict whether more blue or red will be face up. Ask: Has one colour more chance than the other? How do you know? Would it make any difference if we used more counters? What if we just used one counter? (See Key Understanding 6; Collect and Organise Data, Key Understanding 3; ***First Steps in Mathematics: Number***, Calculate, Key Understanding 2, 'Beginning'.)

### Two Steps Forwards

Play 'Two Steps Forwards' using a coin. Ask: Would you expect heads or tails to come up more often? Play the game again with a weighted coin. Ask: Where might you end up after six throws? Why might one side come up more than the other? (See Key Understanding 3, 'Beginning'.)

### Changing the Numbers

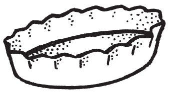
Ask students to consider the effect of changing numbers in dice games that require particular numbers to be rolled. For example, in a 'Make a Bug' (see page 38) game you need 6 to start by drawing a body; then rolling 1 allows you to draw a head, 2 a feeler, 3 a front leg, 4 a back leg, 5 a middle leg. Suggest that students change which numbers are required for which parts. Ask: Would this change your chances of making a beetle or starting the game? Are any of the numbers harder to get than others?

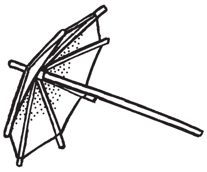
### Roll of the Die

Invite students to decide whether numbers on a die have an equal chance of coming up. Get them to play a game involving dice, such as Snakes and Ladders. Give each student a different number to roll before they can start the game. Ask: Are any of the numbers more difficult to throw? Is a 6 really the most difficult? Why not?

### Choosing Teams

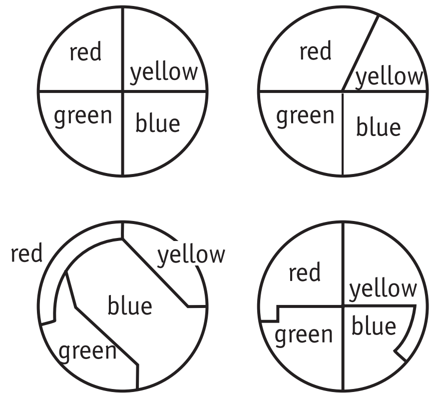
Have students think of ways to decide which of two teams will go first. Ask: Could we toss a bottletop, drink umbrella or drawing pin, instead of the usual coin or cricket bat? How could we test to see if the alternatives were fair? What would we need to do?





## Spinners (1)

Show students a range of spinners with four colours, some with equal sectors, others with unequal sectors. Ask: Which are fair? Why do you think that? Include some with different areas, but equal distances around the perimeter.




## Spinners (2)

Extend the previous activity by asking students to make and design a range of spinners that look different, but have an equal chance of scoring, say, red or blue. Ask: How many different spinners can you design with equal chances of the two colours coming up? What about with three colours? What do you need to think about when you make a spinner so that each colour comes up equally often?

## Round the Twist

After playing the modified version of Twister™ (page 35) ask: How could we make the spinners so that red is as likely as blue to come up? Have pairs of students design a spinner and then justify why their spinner is fair. Draw out how spinners can look different but still provide an equal chance for the two colours to come up.

## Equal Chances

Ask students what their chances are of pulling out a white or black block from a feely bag when one of each colour is in the bag. Ask: Could we put more blocks in the bag but still have equal chances of getting a white or a black? What numbers would not give us an equal chance? If we put in other, different coloured blocks, would black and white still have equal chance?

## Fifty-Fifty

Give students a range of situations to sort into those which are a 50-50 chance and those which are not. Ask them to explain why some are not a 50-50 chance. For example, picking a time at random and seeing if it is daylight or not in Perth in January isn't a 50-50 chance, as Perth has more daylight hours than dark hours. Draw out that we say things have a 50-50 chance if there is no reason to expect one to occur more often than the other. Link 50-50 to the idea of half of 100%.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Outcomes

Challenge students to order the likelihood of events where some or all of the events are equally likely to occur. Have them list and order the outcomes from most to least likely to occur for each of these situations:

- rolling a single dice
- flipping a single coin
- spinning an equally divided spinner.

Ask: Why is it difficult to order the different outcomes in each situation? Draw out that when there is no reason to think that one outcome is more likely than another, then the outcomes are equally likely.

### Weather Conditions

Ask students to list possible weather conditions and place them in order from the least to most likely to occur tomorrow. Ask: Which conditions were easy to order? Which conditions are more difficult to put in order? Why is it difficult to say which one should come before the other? Could these conditions be equally likely? Why?

### Choosing Socks

Have students examine different situations and say why two events are or are not equally likely. For example, from which drawers are you equally likely to take a black or a white sock in the dark:

- a drawer with 22 whites and 11 blacks
- a drawer with 8 whites and 8 blacks
- a drawer with 21 whites and 21 blacks?

### Collections

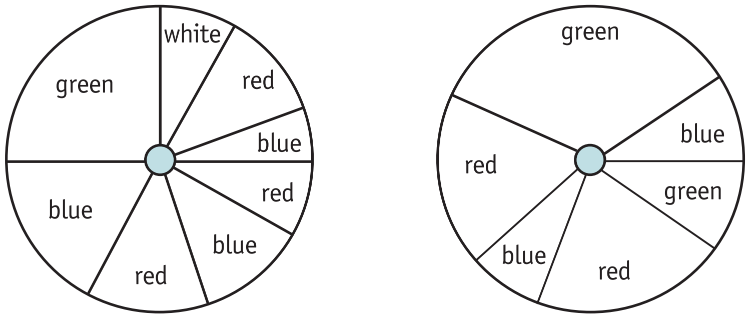
Have students predict the likelihood of drawing out a blue or a red counter from the following collections:

- five reds and 15 blues
- 34 reds and 20 blues
- 18 reds and 20 blues
- ten reds and ten blues
- 40 reds and 40 blues.

Ask: Which ones are easy to decide? Which ones aren't? Which collections are equally likely to come up red or blue? Why? Would it make any difference if we doubled all the numbers? Why? Why not?

## Chance Spin

Have students examine two spinners (see below). Ask: Which one would you use if you wanted to get red? Why is it difficult to choose? Ask them to fix the spinners so that both spinners have the same chance of coming up red.




## Polyhedra Dice Game

Invite students to consider events as less likely, more likely and equally likely to describe the fairness of games. For example: Give each partner in a pair of students their own cubic die. The pair plays a simple track game where one partner moves a space if they roll a 1, 2 or 3, and the other moves a space if they roll a 4, 5 or 6. After a few games, exchange one partner's die with a polyhedra die (tetrahedron, octahedron, dodecahedron or icosahedron). Repeat the game using the same rules. Ask: Is the new game fair? Why? Why not? Which die are you more/less likely to win with? Why? How can you make the game fair? Draw out that fair games require that each winning combination (or winning set of numbers) should have an equal chance of coming up. (Link to Key Understanding 6.)

## Why Is It Fair?

Have students discuss why some games are fair and others are not. Ask pairs of students to draw up a simple track game to resemble a running race. They take it in turns to roll two dice. Runner 1 moves a square if the difference between the two dice is 0, 1 or 2. Runner 2 moves a square if the difference is 3, 4 or 5. Ask: Is this game fair? Draw out that unfair games are where one player's chance of winning is not equal to the chances of the other players.

## Make a Spinner

Have students examine the probability devices (dice, spinners or bags of coloured balls) they made (see page 38) and say which events are equally likely. Have them explain how they constructed their device so that two events were equally likely to occur. Ask: How did you plan your spinner so that it had the same chance of stopping on yellow and blue? Encourage students to compare the different ways they constructed their spinner to ensure that yellow and blue were equally likely to come up. (See Key Understandings 3 and 6.)

# KEY UNDERSTANDING 5

## *We can use numbers to describe how likely something is to happen.*

Just as area is a measure of how big a region is and time is a measure of how long something takes, probability is a measure of chance, or of how likely something is to happen.

We can compare and order objects, spaces and events without reference to numbers, but when we want to say, for example, 'how big' or 'how much bigger' an area is, we use numbers. (See ***First Steps in Mathematics: Measurement***, Understand Units, Key Understanding 3.) Similarly, we use numbers when we want to say 'how likely' or 'how much more likely' an event is. In each case, we use a unit as the basis for quantifying our comparisons.

Events that *cannot* happen all have the same chance of happening, that is, *no* chance, so it makes intuitive sense to say they have a probability of 0. Events that *must* happen are also all equally likely and so it makes sense that they all have the same probability. We have decided to give all events that *must* happen a probability of 1 (or 100%). We then use this certainty of happening as our unit and compare all other events to it in order to quantify how likely they are to occur.

Events that 'might happen' are more likely than events that 'cannot happen' and less likely than events that 'must happen', so we would expect events that 'might happen' to have a probability somewhere between 0 (no chance) and 1 (every chance). It is also reasonably intuitive to think of events that are just as likely to happen as not, as having a probability halfway between 'can't happen' and 'must happen'. In effect, we have developed a numerical scale like this:

| | | |
|---|---|---|
| **certain not to happen** | **as likely as not to happen** | **certain to happen** |
| (impossible—no chance) | | (every chance) |
| 0% chance | 50% chance | 100% chance |
| ☐ | ☐ | ☐ |
| **0** | **0.5** | **1** |

Initially, students should informally and intuitively place events, such as the chance of it getting dark tonight, swimming to Jupiter, doing maths tomorrow, and getting a tail when a coin is tossed, on the scale from 0 (cannot possibly happen) to 1 (must happen). They could compare their placements and debate them, perhaps coming to compromise or 'average' positions. Only *after* they understand the idea of more, less and equally likely (see Key Understandings 3 and 4) should they begin to quantify chance.

Without actually working out probabilities for themselves (but see Key Understanding 6), students in the later primary years should interpret simple probability statements in everyday use. For example, they might say that a 90% chance of rain means rain is very likely and therefore decide to cancel the picnic, but a 10% chance of rain means there is little likelihood of rain and so they are prepared to take the small risk of rain and won't cancel. Given a 50% probability of rain, they might find it difficult to decide. Thus, they should come to understand that events are at their least predictable in the centre of this scale and become more predictable as you move towards either end.

## Progressing Through Key Understanding 5

Initially students can use the scale from 0 to 1 in an informal way, placing everyday expressions of chance such as 'impossible', 'poor chance', 'even chance' 'good chance' and 'certain' on the scale. They have an intuitive sense of the meaning of probability statements such as those associated with weather predictions. As students continue to progress, they understand that probability is the way we measure chance, that is, probability statements give a measure of how likely something is to happen. They understand the 0 to 1 scale and can interpret expressions of probability in general usage such as 'the probability of rain tomorrow is 30%' and 'there's a 50-50 chance'.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### What Will Happen?

Make two labels: 'can't happen' and 'must happen'. Place the labels wide apart. Prepare a set of cards showing various classroom activities, such as using a calculator, playing cards, reading a book and playing with play dough. Invite students to order the cards from most to least likely to happen tomorrow, and place their cards somewhere between the labels. Ask them to say why they put the cards closer to one of the labels, or towards the middle. (See Key Understandings 2, 3 and 4.)

### String Line

Ask students to pin labels such as 'yes', 'might', 'no', or 'always', 'sometimes', 'never' (indicating the likelihood of things happening) in order along a string across the room. They then take turns choosing where events such as 'I'll be at school tomorrow' or 'I'll catch a bus today' should go, by moving a marker along the string line.



### Fifty-Fifty

In incidental discussion of chance events, such as rain, use the term '50-50 chance' to describe events that have an equal chance of happening. Explain it as having an even chance of happening—'maybe it will, maybe it won't'. Draw out that 50 is half of 100.

### Paper Scale

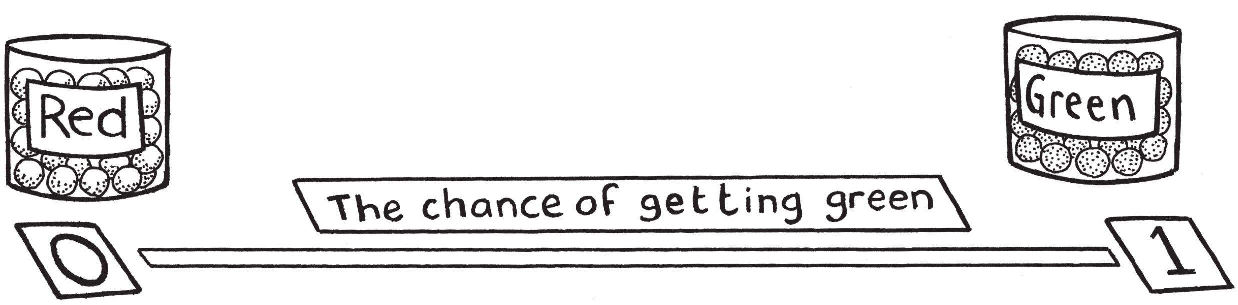
Have students cut a long strip of paper and label one end 'can't happen' and the other end 'must happen'. Ask them to draw pictures of things that could happen after school that day, including unusual as well as routine events. They then place their pictures on the strip of paper, where they think they would be on the scale, and say why.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Coloured Balls

Place a tape across the front of the room, marked with 0 at one end and 1 at the other. Label it 'the chance of getting green'. Show students two jars, one containing only, say, red balls, the other containing only green. Point to the jar containing red and ask: If you drew one ball from this jar, what chance is there of getting green? (None) If we gave that a number, what should it be? Place the jar at the appropriate place (0) on the tape. Repeat for the jar containing green balls, asking: What is the chance of getting green? Place that jar at the other end of the tape (1). Make up another jar containing mostly red and one or two green balls. Ask students where on the tape would show how likely you are to get a green ball. Discuss why. Repeat for various combinations using an intuitive sense of whether you should be closer to the red (0) or green (1) end. Repeat with a jar containing equal numbers of red and green.




### What Will Happen?

After students play 'What Will Happen?' (page 54), introduce the idea of a numerical scale. Link this to measuring other things such as length and time, e.g. if no time has passed we would say there are zero seconds or zero minutes. Ask: If we used the number 0 to show that something is certain not to happen, what number could be used to show that an event is certain to happen? What kinds of numbers might we expect to see on the things that might happen?

### Fifty-Fifty

After students identify a situation where there is a 50-50 chance (link to Key Understanding 3), ask them to identify where this would be on the number scale. Find other situations that would be similarly placed in the centre, e.g. 50% chance of rain, one in two chance of throwing a head, or odds or evens on a six-sided die.

## Middle ✔✔

### Combined Possibilities

Invite students to identify all the possibilities in a simple 'chance' situation. Draw out that one of these must happen or is certain to happen. For example: Show students a jar containing Jaffas™ and Kool Mints™ and ask them to list the possible outcomes if you were to draw out one ball *(a Jaffa™ or a Kool Mint™)*. Ask: How likely am I to get either a Jaffa™ or a Kool Mint™? *(certain to, a sure thing)* So, if we were to give the chance of getting a Jaffa™ or a Kool Mint™ a number, what would it be? *(100%, 1)* What is the chance of pulling out a jelly bean? *(no chance, Buckley's, impossible)* So if we were to give that a number, what would be the sensible number? *(0)*

### More Possibilities

Extend the previous activity to include halfway points. Ask: How could we fill this jar so that you had an equal chance of getting a Jaffa™ or a Kool Mint™? (*equal numbers of both*) What chance would there be of getting a Jaffa™ if you drew out one sweet? Would the chance be more than 0? More than 1? Less than 1? Where between 0 and 1 would it be?

### Rating the Chances

Have students use a 0 to 1 chance scale to estimate the chance of a range of events, such as 'I'll drink some water some time tomorrow' and 'I'm going shopping after school today'. Ask: Why did you decide to put that one close to the 0 and that one close to the 1?

### Prize Box

Encourage students to use numbers informally to rate their chances of events occurring. For example: Ask them what prize they hope to draw out from an enclosed box containing six Cherry Ripes™, three Crunchies™ and one Snickers™. Students informally predict their chance of getting what they want and then indicate where this sits on a scale, with 0 being 'impossible' and 1 being 'certain'. Those who choose the same prize can compare and justify the positions they chose. (See Key Understanding 5, 'Later'.)

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Rating the Chances

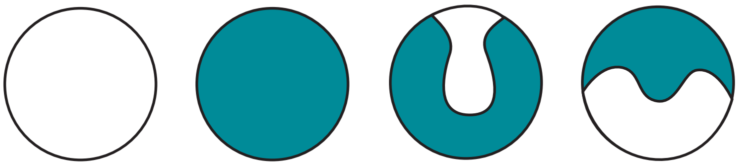
Ask students to use newspapers to help generate a list of national or international events that might happen. Using a number line between 0 and 1, have students informally rate the chances of the events occurring, and explain their reasoning.

### Word Sort

Have students sort chance words such as 'nearly', 'may', 'Buckley's', 'possible', 'doubtful', 'highly probable' and 'fifty-fifty', and place them on a 0 to 1 probability scale according to their description of whether an event will, might or will not happen. Ask them to justify their positioning of words. (See Key Understanding 2.)

### Ordering Spinners

Show students the spinners below. Ask them to put the spinners in order according to which they would rather have, if they needed to spin white to win. Ask: Why did you put them in that order? Have students use a number line between 0 and 1 to informally rate the chances of spinning white, and explain their reasoning for ordering each of the four spinners. (See Key Understanding 3.)



### Pigs Might Fly

Extend 'Pigs Might Fly' (page 39) to have students place the unlikely events they have already ordered, on a scale from 0 to 1. Ask them to justify their order and say why all the events are close to 0.

### Rulak

Have students interpret probability statements as an aid to making decisions. For example: Present monthly probability information about the weather in their home town and an imaginary location (Rulak) in the Northern Hemisphere for each month in the year. Ask students to imagine a friend from Rulak will be coming to visit. Have them write to their friend, suggesting suitable clothes to bring and comparing the local weather to their friend's home, justifying their decisions.

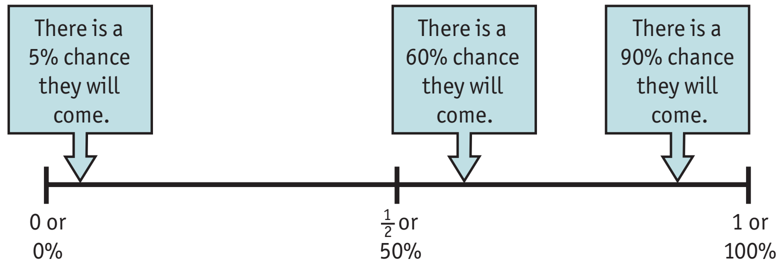
## Later ✔✔✔

### Percentages

After working with percentages as a part of a whole, have students place percentage estimates of probability on the 0 to 1 scale. Reinterpret percentage estimates as approximate fractions. For example: Collect percentage estimates from newspapers, magazines or news programs. Ask: Where on the number line would you place the statement 'The chance of rain tomorrow is 30%'? What does that mean? If the percentages are closer to 0 or to 1, how does that affect our prediction? *(The closer it is to 0 or 1, the surer we are that it will not, or that it will rain; the closer it is to 50%, the less sure we are about rain or no rain.)* (Link to Key Understanding 3.)

### Sorting Percentages

Extend the previous activity and have students sort everyday probability statements in terms of those that make it easy to make a decision and those that do not make it easy. For example: The school sports day is tomorrow. Consider the following probability statements: there is a 90% chance of rain; there is a 10% chance of rain; there is a 50% chance of rain. Which probability statements help you make a decision about whether or not to cancel? Which statement does not help you make a decision? Or: The school canteen manager is deciding whether or not to order more drinks based on whether visitors from a neighbouring school are coming or not. Consider the following probability statements: there is a 90% chance they will come; there is a 60% chance they will come; there is a 5% chance they will come. Which probabilities help you make a decision? Which statement does not help you make a decision? Draw out that events are more predictable at either end of the scale and less predictable in the centre of the scale.




### Prize Box (1)

Have students use numbers informally to rate their chances of events occurring. Invite them to examine a prize box containing six Cherry Ripes™, three Crunchies™ and one Snickers™ bar. The prizes are drawn from the covered box so they cannot see what they are choosing. Have them informally predict their chance of drawing out each of the bars and then indicate where this chance sits on a scale with '0' being certain not to happen and '1' being certain to happen. Students compare and justify their positions they chose for each type of bar.

### Prize Box (2)

Extend the previous activity by asking students to remove one bar from the box. Invite students to again predict their chance of drawing out each of the bars and then indicate where this chance sits on a new scale. Ask: Are your predictions the same as before or have they changed after one bar has been removed? How have your predictions changed? Remove a second bar and then invite students to use another scale to again predict the chance of drawing out each of the bars. Continue drawing out bars and ask: When might you get a 0% chance of drawing out a particular bar? When might you get a 100% chance of drawing out a particular bar? Would the chances change if we replaced the drawn prize after each turn? Why? Why not?

### Predict and Test

Ask students to predict and test the outcome of chance events with and without replacement. For example: Put all the students' names in a hat and have them consider their chances of having their names drawn out to win prizes. After establishing that they have one chance out of the number of names in the hat, invite them to approximate this on a 0 to 1 chance scale. Ask: What is your chance of having your name drawn out? Where would that chance appear on the scale? Draw out a name for first prize, and then ask if and how students' chances have changed for winning second prize. Repeat for third and fourth prize. Ask: What is the chance of the first prize-winner also winning second prize? What affects whether their chance is 0% or the same chance as everyone else? (*It's whether or not the name is replaced after it is drawn out.*) If we continue drawing prize-winners without putting the names back and you end up the very last to be drawn out, what happens to your position on the 0 to 1 chance scale each time? (*The chance moves from near the 0 end at the first draw to 1 before the last draw, when their name will be certain to be drawn out next.*) (See Key Understanding 1.)

### Chance of Red

Have students examine the contents of a box of Smarties™. Ask: If you were to shake them and take one out without looking, what would your chances be of getting a red one? Ask students to estimate this by plotting a point on a number line between 0 and 1. Take one Smartie™ out and eat it. Ask: What is your chance of getting a red now? Has it gone up, stayed the same, or gone down? Where do you think the new point is now? How do you know? Continue until all the Smarties™ have gone, focusing students' attention on the fact that the whole (what 1 represents) changes with each Smartie™ eaten, because the whole chance is the total number of Smarties™ left in the box before each draw. The chance of getting a red each time will depend on the relationship between the number of red Smarties™ not yet eaten and the total number of Smarties™ still left in the box.

## KEY UNDERSTANDING 6

***Sometimes we list and compare all the possible things that could happen to predict how likely something is to happen.***

One of the ways we can make predictions about chance events is to analyse the situation. We study the event, identify what could happen, make some assumptions, and then use reasoning to work out how likely it is that a particular outcome will occur.

The dual notions that 'some events are equally likely' and 'an event certain to happen has a probability of 1' are together needed to analyse situations and make numerical statements about how likely a particular outcome is. Consider tossing a cube that has six different-coloured faces. We 'know' (or assume) that all of the possibilities (red, tan, blue, green, white and black) are equally likely. We are also 'certain' that one of those colours must happen. It therefore makes intuitive sense to think that if each colour has the same probability of showing face up and the six probabilities add to 1, then each colour has a probability of one-sixth.

To be able to undertake such analyses, students need to be able to list all the possible outcomes for an event and make decisions about whether or not the possible outcomes are equally likely. Neither is straightforward for children. Developing the capacity to identify all possible outcomes and to think about whether they are equally likely, should be the major focus of this Key Understanding for primary students, rather than the actual calculation of probabilities.

Early experiences should emphasise 'What are all the possibilities?' rather than 'What is the chance of one possibility happening?' Often students in the early primary years focus on a particular outcome of interest to themselves or that they believe most likely. Through the middle primary years they should develop their capacity to systematically list all of the relevant and possible outcomes for 'one-stage' events (such as throwing one coin or die). It may be considerably later (typically in the secondary years) before they can identify all the outcomes where there are two and three stages or components (e.g. for throwing two coins or two dice).

Initially students may assume that each outcome always has an equal chance of happening. For example, given a four-coloured spinner with unequal sectors, they may assume all colours are equally likely to appear, either not noticing that the sectors are of different sizes or not seeing the relevance. Similarly, having identified the possible results of tossing two coins as two heads, two tails or one of each, they may assume the three possibilities are equally likely. Students will need many experiences analysing possible outcomes, as well as experimenting with the materials, before they will be convinced that all possibilities need not be equally likely.

Some primary school students may learn to describe chance numerically in simple situations involving equally likely events, by counting the number of ways an event can happen out of the total possible events. However, it is likely to be more helpful for children to take the time needed to develop sound basic concepts about chance processes through experimentation and ordering events non-numerically, than to rush to numerical probability.

## Progressing Through Key Understanding 6

Initially students can identify possible outcomes for events that are familiar or that they have observed, but may not systematically list all possibilities. As students continue to progress, when prompted they will list all the possibilities for straightforward situations. Next, students will, unprompted, systematically list all the possible outcomes for a 'one-step' action and use this information to work out numerical probabilities.

As students progress further, they are able to see why the probability that a toss of a fair die will produce a 5 is one-sixth and, if one-quarter of a spinner is red and the rest yellow, why the probability of getting red on a spin is one-quarter and yellow is three-quarters.

# SAMPLE LEARNING ACTIVITIES

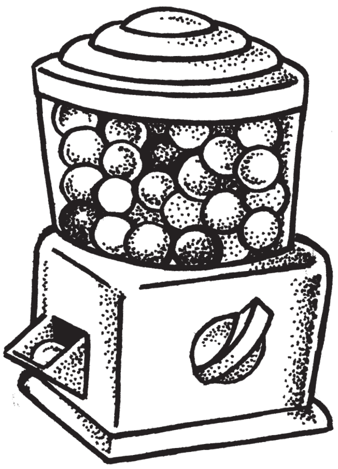
## Beginning ✔

### What Colour? (1)

Invite students to say what colour Smartie™ they might get if they closed their eyes and chose one from small box. Ask: Is it possible that you might get another colour? Which colours might you get? Have you listed them all? How many different colours could you get? Can you get a black Smartie™? Why do you think that? *(Smarties™ do not come in black.)* (See Key Understanding 1.)

### What Colour? (2)

Vary the previous activity to other simple familiar contexts where the possible selections are unambiguous, such as gumball machines, small sets of cards, Pick-up Sticks™ or Lego™. Ask similar questions to draw out the need to list all the possibilities and to also think about what is not a possibility.



### Dice Variations

Ask students to systematically list all possibilities to make checking easier. For example: Give students dice, some with each face a different colour, some with each face a different number, e.g. one numbered conventionally 1 to 6, another labelled 10, 20, 30, 40, 50, 60. Ask: What could you get if you toss this die (e.g. coloured die)? Is it possible to get a red/blue? Is it possible to get an 8 on the 1 to 6 die? Draw out that with the dice labelled 10 to 60, it is easy to say we can't get 55, whereas with the coloured dice we need to check the list.

### Lucky Dip

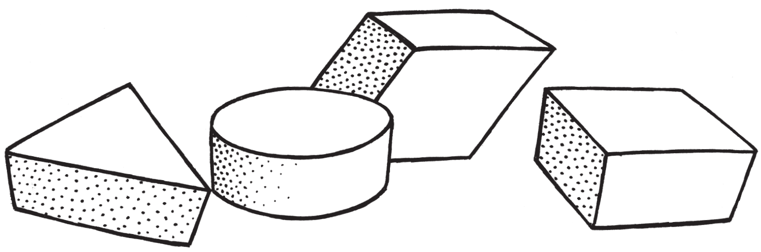
Have students wrap a number of items for a lucky dip game and then write a list of all of the items they have included. They share this list with other students, who then say what prizes they are likely to get. Draw out that some items from the list will now be unavailable.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Blocks

Place four different blocks (triangle, square, circle, rhombus) in a box. Ask students to list the possible outcomes if they drew one out without looking. Ask: Are you more likely to get a triangle, a square, a circle or a rhombus? Why? If you drew out a shape, wrote it down then put it back and then did it again and again and again, what do you think would come up most often—a triangle, a square, a circle or a rhombus? Why? What chance do you have of selecting a triangle? Draw out that there are four possibilities and no reason to think one would be chosen over others. (Link to Key Understanding 3.)



### Selections

Extend the previous activity to other situations where there is only one of each item, and then informally describe the chance of a particular item being selected. For example: Names of ten children in a hat, to be drawn for a job; lucky dips; prizes on an equi-spaced chocolate wheel; day of the week you could be born on.

### Pathways

Provide students with a simple map of a few streets in your location.
On an overhead copy of the map, trace along a route and, as you reach an intersection, ask students to list all the choices a driver would have at that intersection. Make a choice and move on. Have students repeat this activity with a partner, starting at a different spot.

### What Colour? (1)

Extend 'What Colour? (1)' (page 62) by providing each group with a small box of Smarties™. Ensure some boxes contain the same range of colours. Ask students to list the colours in the box and record how many of each colour. Ask them to order the colours from those they would be least likely to get if they closed their eyes and selected one, to the colour they are most likely to get, and to explain their order. They then predict how likely they are to get each colour. Ask: Can you use numbers to say how likely you are to get a red? What numbers do you need to know? *(The number of the colour and the total number.)* (Link to Key Understanding 3.)

## Middle ✔✔

### What Colour? (2)

Extend the previous activity by asking students to compare their responses to those of the neighbouring group. Ask: Are both groups equally likely to get red? ... green? Explain why. Display each group's results for the whole class to see and discuss. Draw out that, because some boxes contain the same range of colours, there might be the same possibilities but the possibilities may not be equally likely (see Key Understandings 3 and 4).

### Tosses

Invite students to select a fixed number of counters with a different colour on each side (e.g. red/blue counters), say eight, and list all the possible outcomes when the counters are tossed together (e.g. eight blue, seven red and one blue, etc.). (See Key Understanding 4; Collect and Organise Data, Key Understanding 3; ***First Steps in Mathematics: Number***, Calculate, Key Understanding 2, 'Beginning'.)

### Which Area?

Identify separate areas of the classroom and number them 1, 2, 3, 4, 5 and 6. Invite students to choose one of the areas to stand in. Roll a die. Students standing in the area whose number was rolled can stay in the game. Show students a range of six-sided dice, e.g. a normal die, one with faces marked 1, 1, 2, 3 ,4, 5 and one marked 1, 3, 6, 6, 6, 6. Ask: Which die would you rather play this game with? Draw out that you have more chance of winning with the weighted dice.

### Card Hands

Use one pack of playing cards to provide groups of students with a 'hand' of seven cards. Ask them to list all possible outcomes when drawing a card from their hand. *(Seven possibilities.)* Ask: Could a person get (name a card) if they drew from your hand without looking? *(Only one group will say yes.)* What chance is there that they will get that card from your hand? *(One group should say 1 in 7, the other groups 0.)* Which suit (or type of card) is someone drawing from your hand most likely to get? *(Groups will have different responses.)* Ask: Why do you think that? So will they definitely get that card?

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Making Selections

Repeat one-stage selection activities such as 'Blocks', 'What Colour?' and 'Card Hands' (pages 63 and 64). Ask students to list all possible outcomes in a table, with frequencies if appropriate, and use the list to make statements about the likelihood of various possibilities. As they gain confidence, help them to express chance statements numerically, including with fractions. For example:

- **What Colour?** Extend and ask students to predict how likely they are to get each colour. Ask: What numbers do you need to know? (How many of each colour and how many in all.) Have students express their probabilities in statements such as *There are 3 chances in 10 that I will get red* or *The chance of getting red is 3/10*.
- **Card Hands.** Extend and have students record the frequency of each suit for their hand (e.g. one group might have 4 spades, 2 hearts, 1 club and 0 diamonds). Ask: Can you use that data to quickly say which suit a person choosing one card from your hand is more likely to get? What chance is there of a person getting a heart? (2 in 7) ... a diamond? (none). (Link to Key Understanding 3.)

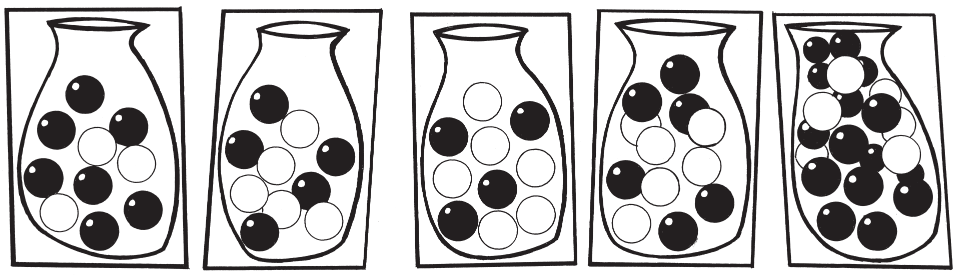
| | |
|---|---|
| Spades | 4 |
| Hearts | 2 |
| Clubs | 1 |
| Diamonds | 0 |

- **Prize Box.** The prize box for the class contains 6 Cherry Ripes™, 3 Crunchies™ and 1 Snickers™. Ask students to use various ways to describe the chance of getting each for a prize. For example: *The chance of getting a Snickers™ is 1 chance out of a possible 10, 1 in 10, 1/10 or 10%*. (See Key Understandings 1 and 5.)
- **Names in a Hat.** Put the name of each student in a hat. Ask students to record how many boys' and how many girls' names are in the hat. Ask: If I draw a name without looking, am I more likely to draw a girl's name or a boy's name? (Link to Key Understanding 3.) What chance is there that I will draw a boy's name?

## Later ✔✔✔

### Frequencies of Balls

Provide students with pictures of bags containing coloured balls in differing proportions (e.g. three red and seven blue, five red and five blue, six red and four blue, six red and six blue, six red and 14 blue), with variations structured to allow various comparisons to be made as described below. Ask students to imagine drawing a ball from a bag without looking. (Link to Key Understandings 3 and 4.)



- Students choose a bag from which they are equally likely to select a red or blue. Ask: What is the chance (probability) of getting a red from that bag? What is the chance of getting a blue from that bag? Are there any other bags for which there is an equal chance of getting a red or a blue? What is the chance (probability) of getting a red from that bag? ... a blue? Is it the same chance as for the other bag? Draw out that 5 chances in 10 and 6 chances in 12 are both one-half. Red and blue are equally likely and nothing else can happen, so each has a half chance of being selected.
- Have students say whether they are more likely to get red or blue from the first bag (three red and seven blue). Repeat for other bags. Sort bags into those where red is equally likely as blue, those where red is less likely than blue, and those where red is more likely than blue. Remind students that when red and blue are equally likely, each has a chance of a half. Draw out that when red is less likely than blue, we would expect its chance to be less than a half, and when red is more likely than blue we would expect its chance to be more than a half.
- Ask students what chance there is of getting red from the first bag (3 chances in 10, or $\frac{3}{10}$). Is this less than a half?
- Reinforce, asking students which of the last three bags they are more likely to get red from (six red and four blue, six red and six blue, six red and 14 blue). Why? Ask: What is the chance of getting red in each case? Do the fractions follow the order of size they expected?

### Make a Spinner

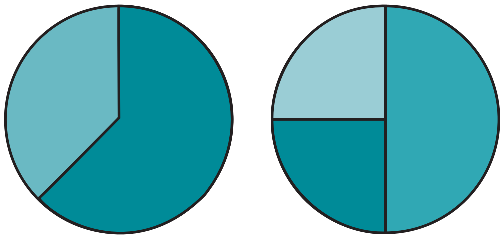
Have students design a spinner so that the chance of it stopping on red is $\frac{1}{4}$, on green is $\frac{1}{2}$ and on blue is $\frac{1}{4}$; OR stopping on blue is $\frac{3}{8}$, on red is $\frac{1}{4}$, on green is one $\frac{1}{4}$ and on yellow is the rest; OR stopping on red is 0.5, on green is 0.1 and on blue is 0. (See Key Understandings 3 and 4.)

## What Colour?

- Extend 'What Colour?' (page 63). Have students colour and draw diagrams to produce specified chances of getting specific colours:
- Provide pictures of bags with five, ten and 15 balls. Ask students to colour each bag so that the chance of getting red is $\frac{2}{5}$.
- Ask students to draw three different bags with a probability of one-quarter of getting red.
- Provide a picture of a bag with ten balls. Ask students to colour it so that the chance of getting green is $\frac{3}{10}$ and getting blue is $\frac{5}{10}$. Ask: If the rest of the balls are yellow, what is the chance of getting yellow? Can you draw another bag with more balls, which has the same chances of getting blue, green and yellow?

## Spinners

Provide different spinners with differently proportioned segments, but with easily perceived fractions, e.g. one-quarter, one-quarter, one-half; or one-third, two-thirds (see below). For each spinner, ask students to list the possible outcomes, discuss whether each outcome is equally likely and say why some outcomes should occur more than others. They then use numerical statements to describe the chance of each outcome occurring.



KU 6 Understand Chance

## KEY UNDERSTANDING 7

***Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future.***

One of the ways we make predictions about chance events is to use data about what has happened in the past, or to carry out experiments (simulations) that we think are sufficiently like the situation we are interested in to act as a kind of proxy for it.

Students should carry out experiments that involve chance processes (e.g. spin a spinner, toss a bottletop) and examine the outcomes. Initially, discussions and predictions should refer to the range of possibilities—what could happen. As suggested for Key Understanding 6, students should learn to make complete lists of possible outcomes from simple 'experiments' (e.g. tossing a die), demonstrate how each outcome may occur, and argue why they think they have the complete list. Having worked out by analysis (as described in Key Understanding 6), what they expect the chances are of, say, a 6 coming up when they toss a die, students should experiment to test their predictions. They will need many experiences before they realise that their prediction of 1 in 6 refers to the long term, not to the short term. Only if we make a very large number of throws, would we expect the actual results to be consistently close to one-sixth.

Students' experimental work should also include many situations that are difficult or impossible to analyse, but are experimentally simple. For example, it is pretty well impossible to work out the probability of a drawing pin falling point up or down by thinking about it. However, it is quite straightforward to throw a drawing pin a large number of times and use the relative frequency of appearance of the two outcomes to estimate how likely each is to occur in future. Working out the chance of future events is often based on the idea that if phenomena, or people, behaved in a certain way in the past, they are likely to behave similarly in the future. For example, if it rained on 250 July days in the past ten years, we can use the ratio 250/310 (rainy days/total days) or about 76% as an estimation of the chance of rain on future July days.

This doesn't tell you if it will rain on a particular July day, or even how many rainy days there will be this July, but it will tell you there are likely to be more rainy days in July than January, when the probability of rain is, say, 3%.

With help, students in the later primary years can use local data to similarly estimate chance. For example, they can use ratios like the number of accidents occurring at school compared to the number of school days over a period to estimate the likelihood of an accident occurring on any one school day. They can examine factors that could affect the data, such as changes in student numbers or playground equipment. We expect variation from year to year but, if there have been no changes in population or in the school environment, we would expect any fluctuations to be within a range.

Where it is impossible, difficult, costly or inappropriate to use past information or generate real data about a situation, a simulation or model can be created that replicates the important aspects of a situation. For example, tossing a coin can replicate a situation that has two equal outcomes occurring randomly, like girl or boy births, so families can be modelled using a number of coin tosses to model a family of that size.

## Progressing Through Key Understanding 7

Initially students will, with assistance, be able to use experimental results to determine a range of possible outcomes and informally use relative frequencies to estimate probabilities.

As students continue to progress, they will be able to plan simple experiments and derive the ratios needed from their data to generate numerical probability statements.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### What's for Lunch?

Have students record the contents of their lunch for a week and then use this to say what they are likely to get each day for the next week. Ask: How did you know what you were likely to get for lunch? Did you ever find something that you didn't expect? How did this happen? (See Key Understanding 3.)

### After School

Ask students to record their after-school activities for a week or so, and then predict what they are likely to be doing over the next few afternoons. Ask: Does the list of what you did last week help you to predict what you might be doing this week? (See Key Understanding 2.)

### Weather Watch

During summer or winter, ask: Do you think it is likely to rain today/tomorrow? Ask students to record their predictions and then record the weather over a couple of weeks. They then review the data to see what the weather has been like and use this information to predict the weather each day for the next week. Ask: How did the data help us to know what the weather might be? (See Key Understanding 2.)

### Tosses

Ask students to toss several red/blue counters and try to get all counters to land with one colour face up. Have them record the results of each toss. After a few tosses, ask students to predict whether they are likely to get two blues, two reds or similar, on their next toss. They then toss and record a few more times. Ask again: Are you likely to get two reds on your next toss? (See Key Understandings 1, 4 and 6.)

### Team Sports

Invite students to look at the teams that have won the schools sports carnivals over the past few years and predict which team will win this year. Ask: What makes you think this team might win? Have the teams changed since last year? Can we be sure which team will win?

### Games

Have students play board games that require a particular number to come up on a die, e.g. Make a Bug (page 38). Afterwards, ask: Can you make sure that the number you want will come up? Does the position of the die before you throw it make a difference? Does wishing for your number help? Have them test out each suggestion and record their throws to see if it helps the number to appear. (See Key Understanding 1.)

### Safety First (1)

After students have played on climbing equipment, ask them if they felt unsafe on any parts of the circuit and why. Have them reflect on how the equipment was set out, what they were doing, how many others were using it at the same time and why they felt unsafe. Invite students to suggest ways to alter the design of the circuit and try it the next day.

### Safety First (2)

Extend the previous idea to other activity areas, e.g. painting corner, home corner, blocks, carpentry, cooking, water play, sandpit. Use this information for students to contribute to the classroom rules for each area. When they are playing in an unsafe way, ask them to recall the rules, and say what might happen if they continue.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Which is Harder?

After students have played a dice game requiring a 6 to start, ask: Are sixes really harder to get than other numbers? Would changing the starting number to a 2 be better? Have students work in pairs to find out, by throwing the die 20 times and producing a tally against the list of six possible outcomes. Ask them to describe what they found. Ask: Who found 6 came up less often than 2? Who found 2 came up less often than 6? Who found they were the same? … close to the same? Have each pair combine their results with two other pairs to produce results for 60 throws. Write the results on the board. Discuss differences and similarities in data. *(More variation in 20 throws, reduced variation in 60 throws.)* What does the data suggest? Are all numbers equally likely or are some more likely than others? Total the scores for the whole class so they now have 200 to 300 throws. Ask: What does this suggest to you? Is 6 less likely to come up in the long run? Why does it feel that way? *(Partly that there are five times as many chances that six will not come up, partly because they are anxious to get started!)*

### Triangular Spinner

Repeat the previous activity for several other simple random devices, such as an equilateral triangular spinner showing three different colours.

### Dice Patterns

During games involving dice, ask students to record each number they throw. Afterwards, look at the sequence of numbers and see if there are any patterns. Ask: Can writing down this sequence help us to say what the next number will be when we throw a die? How often did each number come up? Will each number come up the same amount of times if we play the game again?

### Two Steps Forwards

Have students toss a bottletop, drawing pin, drink umbrella or a button. When it lands face up, they take two steps forwards; when it lands face down, they take two steps backwards. They can play with a partner and see who is first to reach a given line. Have students record the outcome of each toss and use this information to decide if it would it be better to take forward steps when it lands face down instead. Ask: Which side comes up more often? Is it likely to continue to come up more often? (See Key Understanding 4.)

### Testing Predictions

Play the previous game again, with students carrying out an experiment (as in 'Which is Harder?', page 72) to test their predictions about one of the devices they used, e.g. bottletops, drawing pin, drink umbrella or buttons.

### Fair Dice

While playing dice games, invite students to compare a regular six-sided die with one made from a rectangular prism. Present this scenario: Rebecca says we shouldn't use the rectangular prism because some numbers won't have as much of a chance of coming up. Beau says it doesn't matter which die we use, the numbers are all still equally likely to come up. Ask: How can we find out? Students carry out an experiment (as in 'Which is Harder?', page 72) to test their predictions.

### Footy Tipping

Conduct a class footy tipping competition. Have students look at each team's previous wins and losses when making their predictions for the next round. Ask: Does knowing how often this team has won before help you? How? (See Key Understandings 1 and 3; Summarise and Represent Data, Key Understanding 5.)

### Heads and Tails

Invite students to predict the outcome of tossing two coins by standing up and placing two hands on their head (two heads), two hands on their bottom (two tails) or one hand on each (one head and one tail). Toss two coins. Students who predicted correctly continue to play. Keep a record of what was thrown each time and ask: What do you think you should pick to give you the best chance of staying in the game? Ask them to justify their decision.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Making Selections

After completing the 'Making Selections' activities on page 65, have students test their predictions. Build up data sets starting with pairs, then groups, then the whole class, as for 'Which Is Harder?' (page 72), and ask similar questions. For some of these examples, the possible outcomes may not be equally likely. Draw out that for small amounts of data such as the 20 collected by pairs, results are quite variable and one set of 20 would not be enough for you to be confident. For 60 bits of data there is less variability and results may be more convincing. Several hundred bits of data collected by the whole class give a long-term pattern more consistent with expectation. Also draw out that if the short-term pattern does not seem to fit our conclusions from analysis, we might not be concerned, but if the long-term pattern does not fit we might go back and check the thinking we used that led us to our prediction. (See Sample Lesson 4, page 78.)

### More Testing

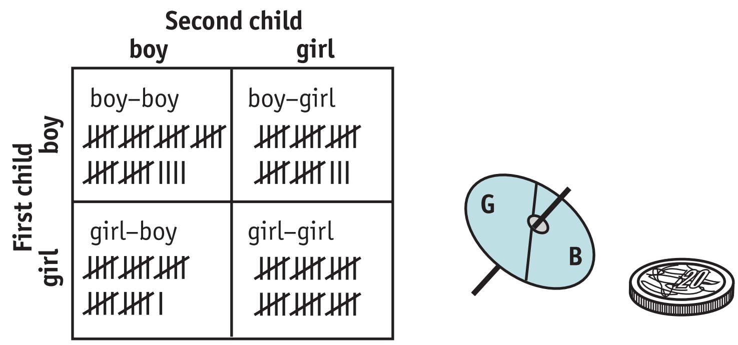
Repeat the previous testing for other predictions made in relation to Key Understanding 6.

### Irregular Spinner

Provide students with a spinner that has three obviously unequal sectors, differently coloured. Ask them to list the possible outcomes of a spin, and order from least likely to most likely (without numbers). Have students hypothesise about the frequency with which each colour will appear. They experiment to test their prediction, building up from pairs to groups to the whole class, as for the previous activity. Ask students to describe their data, e.g. *About half the time the green came up; the red came up about twice as often as the blue*.

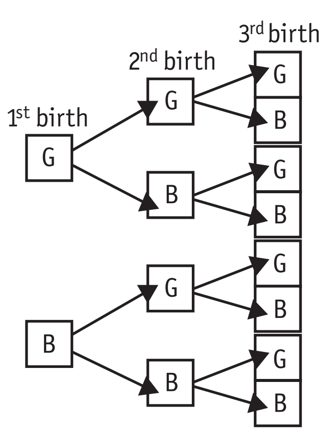
### Families

Have students hypothesise about the likelihood of a family with two children having two boys, two girls or one of each. Ask them to suggest different ways they could represent (simulate) a birth with an equal chance of being a boy or a girl, e.g. toss a coin and make it heads for a girl and tails for a boy, or use a spinner with half marked B for a boy and half marked G for a girl. Toss or spin once for the first birth, and then again for the second birth, or use the coin for one birth and the spinner for the next birth. Invite students to experiment to test their prediction, building up from pairs to groups to the whole class, as for the previous activity. Have students record their experimental results in a two-way table and use it to describe their data, e.g. *About half the time there was one of each; about a quarter of the time there were two girls; and about a quarter of the time there were two boys*. (Link to Collect and Organise Data, Key Understanding 5.)




### Bigger Families

Have students extend the previous activity to families with three children, using the spinner or coin three times to simulate the gender of the three births. Ask: How will you record the results? Why won't a two-way table work? What fraction of three-child families do you think might have all boys or all girls? What fraction do you think will have two boys and a girl, or two girls and a boy? Have students use their spinners or coins to produce a large number of 'families' and then help them use the data to predict the fraction of three-child families that are likely to have same gender children and the fraction that are likely to have either two boys and a girl, or two girls and a boy. (The diagrams below may help students understand the variation in probability and record the results of their experimentation more systematically.)




| |
|---|
| GGG |
| GGB |
| GBG |
| GBB |
| BGG |
| BGB |
| BBG |
| BBB |

### Testing Bigger Families

Students could test the result of their predictions in the previous activity by collecting real data in the school. Negotiate with other classes to find all the families of three children and record how many boys and girls they have, and the order of the births. Ask: How well do the fractions of three-child family combinations in the school match your simulated families?

## Later ✔✔✔

### Race to 50

Have students play a dice game called 'Race to 50', by tossing two dice and adding the total, then using their calculator to keep a cumulative total. After the game, ask: What numbers can you get from each toss of the two dice? (See Key Understanding 6.) Have students list all of the possible totals and toss the dice to see how often they get each number. Ask: Did you get one total more often than another? Why do you think that happened?

### Weather Watch

Ask students to get weather data for several years from the Weather Bureau about various conditions such as sunny days, rain, fog early in the year, and so on. Use it to calculate probabilities for the different months in the year. Keep corresponding data throughout the year and compare the statistics from each month with the probability predictions.

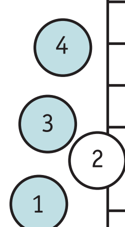
### Counters (1)

Have students take four cardboard counters, each of which is red on one side and blue on the other. Have them list the possible results after the counters are tossed in the air (that is, four red; three red and one blue; two red and two blue; one red and three blue; four blue). Invite them to predict whether these results are equally likely and if not, to say which results they think are more likely than others. Have them experiment by throwing and recording the results each time, building up the quantity by combining results in pairs, groups, then the whole class, as in 'Which Is Harder?' (page 72). Ask: How do the results match your predictions? Students should find that the five different outcomes are not equally likely.

| | |
|---|---|
| 4 B | 𝍸 \|\| |
| 3 B, 1 R | 𝍸 𝍸 \|\|\|\| |
| 2 B, 2 R | 𝍸 𝍸 \|\| |
| 1 B, 3 R | 𝍸 𝍸 \|\|\| |
| 4 R | 𝍸 \| |

## Counters (2)

Extend the previous activity by having students record exactly how each of the counters falls on each toss. Invite them to number the counters 1 to 4 on each side, and then use a grid to record the colour of each counter after each throw. As before, have students combine the data to obtain several hundred results. Invite them to use their recordings to work out why an all-blue result or an all-red result is less likely than a two-blue and two-red result. Ask: How many different ways did the counters show two red and two blue? How many different ways did the counters show all red? Draw out that there is only one way the four counters can fall to be all red, but any of the following six different arrangements results in two reds and two blues: RRBB, BBRR, BRRB, RBBR, RBRB, or BRBR.




| 1 | 2 | 3 | 4 |
|---|---|---|---|
| R | B | B | R |
| B | R | R | B |
| B | B | B | R |
| B | B | B | B |
| R | B | R | B |
| B | R | R | B |
| R | R | R | R |
| R | R | B | B |
| R | B | R | R |
| | | | |
| | | | |
| | | | |

## Holidays

Have students gather local weather data about the average number of wet days in April. Help them to formulate questions, e.g. What is the likelihood of no rain over the five-day Easter break (all in April)? Ask students to make a spinner to represent the proportion of rainy and dry days in April. They experiment by spinning five times and recording whether all days were dry or not. Repeat ten times with a partner recording. Swap and combine results, giving 20 trials. What fraction of simulated five-day periods had no rain? Combine results for three pairs. Compare group results. Combine results for whole class. Discuss this as an estimate of the chance of five particular days in April being dry.

# SAMPLE LESSON 4

**Sample Learning Activity:** 'Later'—Making Selections, page 74

**Key Understanding 7:** Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future.

*To make the process more efficient, we put the coloured dice into large foam cups and taped a sheet of clear plastic firmly over the opening. The students tipped it upside down, shook it several times, then turned it right way up and stood it on the desk. They could then look through the clear cover to check which face was showing. With this easily constructed device, a large number of results could be generated in a short time without the noise or confusion of throwing dice around.*

## Teacher's Purpose

I had been helping my class of 11- to 12-year-olds extend their ideas about equal likelihood (see Key Understanding 4), using dice that had different-coloured faces: two red, two blue, one yellow and one green. They could list the four possible outcomes and most agreed that red and blue should have the same chance because they each had two faces. Some also recognised that yellow had the same chance as green, but each had less chance than red or blue. However, they could not agree about what this meant or predict the results of actually throwing the dice.

## Challenging Their Ideas

I decided to organise a way of recording the results of their dice throws that would challenge their ideas about what it is we are estimating or predicting when we say that, for example, red has a 2 in 6 chance.

I provided some strips of grid paper six squares wide and asked students to label the columns according to the colour of each face. In pairs, students threw their die, colouring the squares to match as each result came up. (They had one strip between two, and took turns throwing and colouring.)
I initially asked them to predict what their page might look like after 12 throws. Some expected the results to show about four red, four blue, two yellow and two green. Others thought they'd be about even for each colour, while others insisted anything could happen: *It's just the luck of it, anything could come up so you just can't say.*

Each pair threw 12 times and compared their results with what they thought would happen. Many were surprised at the results, which seemed to confirm the opinions of those who believed it was just luck and you couldn't predict the outcomes. The results of the pairs were quite variable. The students drew a thick line outlining the totals to date (see diagram), to keep a record of their result after 12 throws.

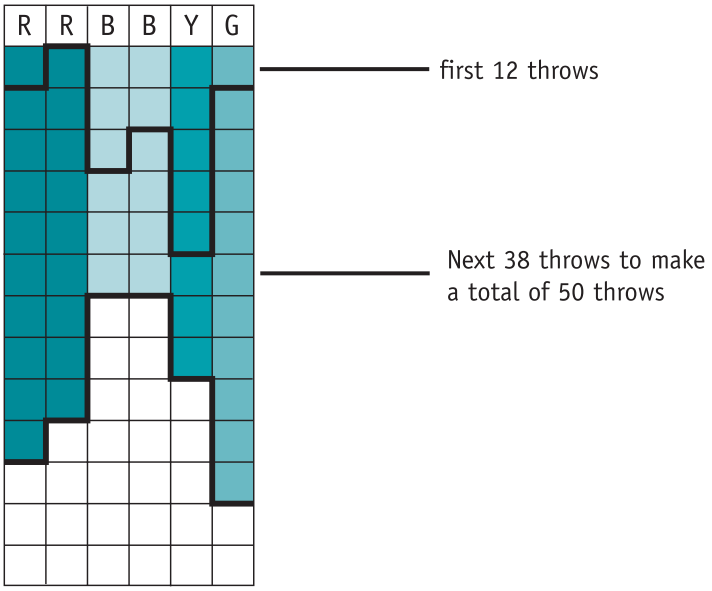
I then asked them to predict what might be the result of 50 throws. They were less keen to predict balanced results this time and most thought we could end up with anything. However, some thought that the columns might even up. After each pair had obtained 50 results they again outlined the shape and we pinned their strips to a board for the class to look at. The variation in the results seemed to further confirm for many that there was no way to predict the outcomes. *They're all different, you just can't tell.*

*So what might we mean when we say red has a 2 in 6 chance?* I asked.

*I think it might be that you* ***should*** *get two reds when you do six throws,* said Sandy, *but it's not going to happen like that really.*

*What do you mean?* I asked.

*Well I thought it's like, in six throws it's supposed to be two red because there's two red faces, but really it doesn't happen like that, they're all different.*




I asked them to look across everyone's results and see if they could see any patterns. A variety of observations were made, but Christalla noticed something important. *There's two reds in every six throws for a lot of rows in the 50 results, but hardly any in the 12 results,* she said. This opened up an opportunity to build further on this idea. *I wonder what would happen if we threw some more? Would we expect to get more finished rows?*

Over the next day, students added to their columns, being careful to record every result. Each pair recorded about 200 results and students saw that they all had a lot more completed rows of two red, two blue, one yellow and one green, as well as a lot of 'left-over' results. I then suggested we put all of our results together to see how they looked overall.

## Drawing Out the Mathematics

The pairs cut off their 'left-overs', then all pasted their completed rows on the display board, one under each other in columns. They then cut up the 'left-overs' results into strips of the different coloured columns and added them on to the display, matching them up, somewhat like a jigsaw, to make as many completed rows as possible. When all had been combined we found we ran out of red first, leaving extra blue, yellow and green.

As a class we looked at the display and talked about what it told us about the idea of a 2 in 6 chance of red. In discussion we drew out the following observations:

*Although few primary school students understand ratio numbers, the visual effect of combining the recorded outcomes in this way enables them to see what is meant by, for example, a 'two out of every six chance'. They can see visually how much variation there is after a few results, but also the result of collecting a very large number of outcomes.*

- There weren't many people who got even one complete row in the first 12, there were some more who got complete rows when we did 50, and a lot more still in 200.
- After 200 throws there was still a lot of variation, but everyone ended up with a lot more reds and blues than yellows and greens.
- When we put all the results together we ended up with more than 3000 throws, but we didn't get exactly two reds for every six throws—there were still a lot of left-overs. We would have needed to get about 100 more reds to make it even.
- We could say that for the 3000 results there were, 'on average' about two reds for every six throws.

I was satisfied that my students now had a better intuitive feeling about what we mean when we express a probability as a ratio. They were beginning to understand that this was an indication of the pattern we expected in the very long run, not what we would expect in the short run.

CHAPTER 4

# Collect and Process Data (Part A)

## Collect and Organise Data

This chapter will support teachers in developing teaching and learning programs that relate to Part A of the outcome:

***Plan and undertake data collection and organise data for effective interpretation***

### *Overall Description*

Students systematically collect, organise and record data to answer their own questions and those of others, e.g. 'Which school lunch is best liked?', 'Which animal is most scary?', 'What shapes and proportions do people like best?', 'How does absenteeism from school relate to the time of year?', 'How much water is used in the school each year and for what?', and 'Does having a part-time job affect school results?' They clarify and refine questions and plan surveys, experiments and simulations to help answer them in unbiased ways, considering both the data collection instruments and the size and nature of samples.

Students understand that:

**1** classification underlies the organisation of data

**2** how we classify depends upon the questions we want to answer

**3** the way the data is organised can illuminate or mask certain of their features

**4** this influences how the data is interpreted and used.

For example, one classification of sports preferences might suggest that students prefer ball games; another might suggest that balls are not relevant, rather that students prefer team to individual sports. They, therefore, realise that data can be distorted accidentally or deliberately to reach inappropriate conclusions. They consider the impact of technological change on the collection and handling of data and of the issues this raises about matters of privacy and social monitoring. They also consider ethical issues in the collection and organisation of data and act responsibly in this regard.

| Markers of Progress | Pointers<br>Progress will be evident when students: | |
|---|---|---|
| Students participate in classifying and sequencing objects and pictures and, with guidance, pose questions about them. | • describe a likeness between several things, e.g. *All of these are leaves*, *The trees and the pencils are alike because they are long and round*, or a difference between several things, e.g. *Some are leaves but some aren't*<br>• classify things using one or two familiar criteria, e.g. leaves by shape and colour, list class members' | names under Indigenous kinship group (or parents' country of origin)<br>• place objects into sequences, e.g. order leaves according to criteria such as darkness or length<br>• suggest how they can answer questions about their collections, e.g. *If I line up the red and green shapes, I will be able to see if there are more green* |
| Students realise that we can answer some questions ourselves by collecting, classifying and sequencing data, and apply unambiguous and familiar criteria consistently when classifying and sequencing. | • pose questions suggested by collected data, e.g. having gone on a 'shape walk', they might ask what shapes occur most often in built things<br>• make predictions related to familiar things, e.g. *We think that dogs are the most common animal kept as pets; We think sixes are hard to get when you throw a die*<br>• in answer to the question 'How could we find out?', suggest collecting objects or information and offer suggestions about what data to collect, e.g. suggest tossing a die to test whether sixes are hard to get | • offer suggestions about how to classify objects or information, e.g. having asked class members what pets they have, suggest ways to classify the animals to test their prediction that the most common type will be dogs<br>• apply unambiguous and familiar criteria to sequence data consistently, e.g. order the children in their age group from oldest to youngest<br>• organise data by classifying items in categories they have created, e.g. make up their own food headings and under each food, list who chose it for lunch |
| Students contribute to discussions to clarify what data would help answer particular questions, and take care in collecting, classifying, sequencing and tabulating data in order to answer those questions. | • suggest information to collect to answer particular questions<br>• clarify questions to decide what data to collect, e.g. *We want to know what are the most popular pets in the class, and we mean best liked, not what most children have*<br>• specify how frequencies or measurements are to be made, e.g. do we count the tank or the fish as 'one' when we're counting our pets; specify that arm length is from inside the armpit to the wrist | • suggest a suitable way to classify data to answer straightforward questions, e.g. suggest sorting their 'coming to school' sketches into groups by transport type (car, bus, bike or on foot)<br>• improve their descriptions of categories to clarify what the category includes or excludes, e.g. when classifying drawings of 'things that scare me' into 'alive' and 'not alive', deciding where 'ghosts' belong<br>• record frequency data carefully using simple formats based on tallies or organised lists, and take care with their measurements |
| Students collaborate with peers to plan what data to collect and how to classify, sequence and tabulate it to answer particular questions, and see the need to vary methods to answer different questions. | • suggest what data to collect to help estimate numbers or quantities, e.g. to estimate how many raisins one could expect 'on average', collect data from a number of small packs of raisins<br>• revise a survey question so it can be answered by 'yes/no' or a simple multiple choice, e.g. begin with 'Which of these colours do you like?' and, after trialling, revise to 'Which pair of colours would you like best for our logo?'<br>• design a test of their predictions about a probability device they have designed, e.g. | a spinner that will come up red most often<br>• construct and use their own categories to answer specific questions, e.g. for 'How animals move', they classify animals by 'walk', 'fly', 'wriggle', and 'swim'<br>• suggest ways to improve a classification to better answer a question<br>• realise that different classifications may tell different things and suggest an alternative classification to answer new questions |
| Students collaborate to plan and refine survey questions and other observation methods for one-variable and two-variable data, and collect and record data, including in databases that are planned with help. | • collaborate in developing and trialling two or three questions involving 'yes/no' answers, simple multiple choice responses, or categories, e.g. 'Do you think teenagers should have to help around the home for pocket money?' (yes/no)<br>• collaborate in making and refining data collection sheets involving lists or tables or scales, e.g. decide which categories to use in investigating animal behaviour for a science project<br>• collaborate to clarify terms to help ensure that data are collected consistently, e.g. in considering 'Do students do more homework as the year goes on?', decide what they mean by 'doing homework' and if 'how much homework' means time spent or some other measure | • improve the collection of measurements to make them more consistent, e.g. decide how to mark off a pace so that each one is done the same way<br>• carry out data collection consistently and accurately, e.g. ask the planned question in the same way each time; measure between the same body parts; round in the same way<br>• plan data collection sheets for the collection of two-variable data, e.g. the amount of time spent watching TV and reading for each student surveyed<br>• enter data in databases with fields already defined, e.g. enter information on the books each class member reads in preparation for a Book Week display |

## Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

| Key Understanding | Stage of Primary Schooling—Major Emphasis | KU Description | Sample Learning Activities |
|---|---|---|---|
| **KU 1** We can answer some questions (and test some predictions) by using data. | Beginning ✔✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 84 | Beginning, page 86<br>Middle, page 88<br>Later, page 90 |
| **KU 2** We can produce data by: counting or measuring things, asking groups of people, watching what happens, or reworking existing data. | Beginning ✔✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 94 | Beginning, page 96<br>Middle, page 98<br>Later, page 100 |
| **KU 3** Organising data in different ways may tell us different things. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 102 | Beginning, page 104<br>Middle, page 106<br>Later, page 109 |
| **KU 4** We should make our data as accurate and consistent as possible. | Beginning ✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 118 | Beginning, page 120<br>Middle, page 122<br>Later, page 124 |
| **KU 5** Sometimes we collect data from a subset of a group to find out things about the whole group. There are benefits and risks in this. | Beginning ✔<br>Middle ✔<br>Later ✔✔ | page 130 | Beginning, page 132<br>Middle, page 133<br>Later, page 134 |

**Key**

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

## KEY UNDERSTANDING 1

### *We can answer some questions (and test some predictions) by using data.*

There are many ways that we answer the questions we have about the world. One way is to ask someone we believe to be an authority *(Mummy, why does it get dark at night?)*, another is to refer to a textbook (perhaps an astronomy text). However, many questions can be answered by the production of data. Using data to answer questions is the essence of 'the scientific method'.

In developing this Key Understanding, students should learn that:

- we (and others) can answer many of the questions we have about the world by referring to data
- not all questions can be answered by referring to data
- some questions cannot be directly answered by data but can be reframed into questions that can be
- some questions cannot be completely answered by data, but data may contribute to the answer
- questions often have to be made more clear or precise before we can decide what data is needed
- we must make sure that the revised questions still get at what we wanted to know in the first place
- sometimes our questions can be answered by data we produce ourselves (primary data collection)
- sometimes our questions can be answered by data produced by others and/or already available (secondary data collection)
- sometimes data we (and others) have already collected suggests new questions.

They should also learn that:

- predictions are not simply guesses, they are 'best guesses', 'informed guesses' or 'judgment calls' based on our previous experience and knowledge, and our theories and analysis

- when we test a prediction we formulate a question or hypothesis and produce data to answer this
- when we test a prediction we answer the specific question 'Will what I predict, happen?'

Although young children ask many questions, often they do not consciously think of producing data as a way of answering these for themselves. It is often possible to build on the questions students spontaneously ask so that they learn to think of data collection as an appropriate question-answering strategy. However, many of their questions are not simple enough to be readily answered by the kind of data they can produce. Thus, teachers may need to model the posing of simple questions about collected objects or pictures, directly prompt students or help them to ask more searching questions.

Sometimes the question itself makes it clear what type of information is needed (see Key Understanding 2), but many questions and predictions are framed in rather general terms. For example, 'How safe is the playground?' cannot be answered directly by data, since students are unlikely to have an obvious or immediate measure of 'safety' (although health and safety organisations may). In this case, we have to ask ourselves whether we can reframe the question into one that can be answered by data, or alternatively we need to produce data on something that we think would be an indicator of safety. Students should experience the processes involved in posing questions for themselves and refining and reframing them to make them accessible to data that they produce afresh for this purpose or which is already available.

## Progressing Through Key Understanding 1

Initially, with guidance, students pose simple questions about things they can observe such as, *What type of fruit is there most of (on the table)?* and participate in discussions about how to find out. As students continue to progress they realise that they can answer some questions by collecting data. They are able to offer some appropriate data-oriented questions and make predictions such as, *I think the most popular fast food will be pizza*. Next, when prompted, students will attempt to clarify and refine their questions to decide what data to produce *(What do we mean by 'most popular pet'?)*. As students progress further they can use these skills on practical problems that may not be obviously mathematical, and will collaborate to develop sub-questions that contribute to addressing a general concern.

# SAMPLE LEARNING ACTIVITIES

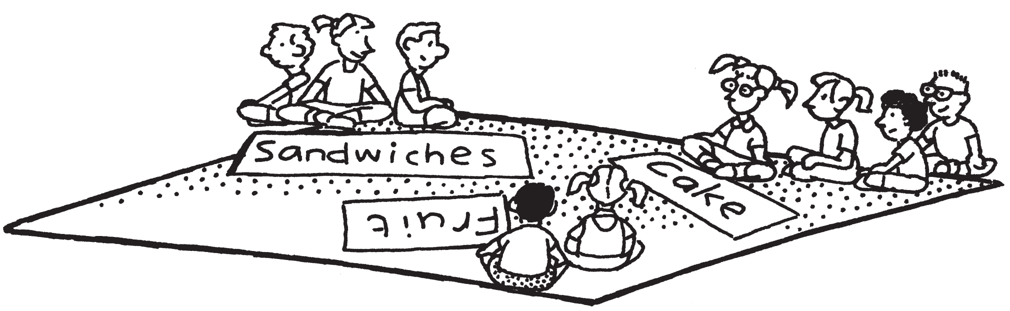
## Beginning ✔✔✔

### Yes/No

Hang a card with 'yes' on one side and 'no' on the other next to individual name cards. Display a question above the names, e.g. 'Have you had fruit?' or 'Do you need to change your book?' Have students turn their card to the appropriate side to show the answer relevant to them. (See Interpret Data, Key Understanding 3.)

### Picnic Plan

Pose the following question: What food should we take on our picnic? Write each suggestion on a card and position them around the mat. Ask students to sit next to their choice and then count and record the number at each card. Remove the least popular and continue until there is an appropriate result. Repeat this activity using other questions, including those that arise spontaneously in the classroom.




### Finding Answers

Respond to questions students spontaneously ask by suggesting data collection. For example: *Are there going to be enough brushes for everyone?* Ask: Do we have to rely on guessing? How could we find out? Draw out that collecting data enables us to answer questions.

### Questions, Questions

Model the posing of simple questions. For example, after lunch, ask: Do you think most of us had sandwiches for lunch? What type of fruit do most people like? How many different sorts of biscuits do we eat? Next, ask: how could we find out if most of us had sandwiches for lunch? Do you think that is the same for the class next door? How could we find out? Draw out that these are questions that could be answered by collecting information. Ask students to volunteer some more questions. Discuss with the class whether they can be answered by collecting information. Revise the questions if necessary. Ask students to make up a question of their own that their partner could answer by collecting information.

### Prompting Questions

Prompt students to find questions that they can answer by using data. When working on a theme such as emus, ask: What do you wonder about emus? Which of these things could we write a question about? Which of these questions could be answered by counting things? Which could we read about to find out? (Link to Key Understanding 2.)

### Searching Questions

Model the process of extending simple questions into more searching questions. For example: 'Are the boys or girls in our class taller?' could be extended to 'What if we were in a Year 6 class? What if we were talking about Year 12 girls and boys? What if we were asking about adults? Does age make a difference?'

### Clarify and Refine

After students have framed a general question, model questions they could ask themselves to clarify and refine their questions. For example: 'Is football the most popular game for children?' could be refined by asking, 'Do we mean the game that is liked the best or the most commonly played?'

### Grandpa's Breakfast

Encourage students to decide which questions can and can't be answered by collecting data. After reading *Grandpa's Breakfast* (Croser, 1997), ask students to decide what they would like to know about breakfasts. Ask: Which of these questions could we answer by collecting information? Can we rewrite some of our questions so that we could answer them by collecting information? (See Key Understanding 2.)

### Making Predictions

Have students make predictions based on experience when growing plants in science. Ask them to predict which plant will grow the fastest and decide what data they need to collect to test their prediction. Ask: Why did you predict that plant would grow the fastest? What question does this particular data answer?

### Story Time

Before reading a book to the students, ask them what they would like to know from the book. List the questions, then have students work through the list and say which could be answered by counting something, and which by recording other information from the book.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Question Box

Have students replace general questions with more specific ones that can be answered using data. For example, ask them to decide which questions from a class question box they would like to answer. Ask: Could we answer this question by collecting data to answer a different question? To find out why dogs bark, we can't survey dogs, but we could ask dog owners what is happening at the same time as their dog barks. (See Key Understanding 2.)

### Counting on Frank

After reading *Counting on Frank* (Clement, 1990) to students, ask them to think of questions the boy in the story might have been asking himself for each calculation. Have students then pose their own questions that can be answered in a similar way.

### Refining Questions

Have students refine a question so they can use data to answer it. During technology lessons, invite students to decide which questions to ask in order to evaluate their designs. For example, after they have designed paper planes from different materials, their initial question might be: *Which are the best materials to use?* Help them to rework the question into: *Which plane flies the furthest?* Ask: Does the second question really answer the first question? What else might be considered as a measure of the best design?

### New Questions

After collecting some data, have students pose new questions. For example, after reading one book about glaciers, ask: Do glaciers really move at 1 m per year? What information would we need to collect to answer that question? After students produce data (from reference material) to answer the initial question, ask: What else could we find out from the data? What other questions could we ask? Does this data help us or do we need to collect some more data to answer the other questions? (Link to Key Understanding 2.)

### Using Data

Encourage students to decide whether they could answer questions by collecting data. For example, at the beginning of a unit of work on pets, ask students to brainstorm things they would like to find out about the topic—Which pets are most popular? What should you do if your pet is sick? Ask: Could we answer any of these questions using data? What questions might you have to change to collect data to answer them? For example, what do we mean by 'most popular'? How could we check this?

### Recycling

After collecting data to answer a question about recycling in their community, ask students whether the data answers their original question. Ask: What did you find out by collecting the data? Has it helped us answer our question? Has it answered a part of our question?

### Model Cars

Ask students to use predictions to frame questions. For example, after making model cars in technology lessons, have students predict which car will travel the furthest down an incline and then decide what data they need to collect to test their prediction. Ask: What question is your prediction testing? They may modify this question so that it directly suggests what data to collect. (See Key Understanding 4.)

### Early Birds

Have students reframe questions so they can collect data to answer them. Students collect popular sayings and ask a question about one. For example: What does 'the early bird catches the worm' mean? How could we rework the question so that we could understand it and find out if this is true? Draw out how the question could become 'Do people who wake up early get more done during the day?'

### *Did You Know?*

While some young students may not know the difference between a statement and a question, for many, the way a question is formulated may be culturally specific. "*An acceptable way of seeking information in some Aboriginal communities is to hint at what you want to find out through a leading statement, leaving the person you are seeking information from to choose when, how and whether he or she will tell you more …*" For example, "*rather than directly asking What part of the book did you like the most?', Aboriginal people might be more inclined to seek information through a trigger statement like 'I reckon the best bit was when Joey hid under the tank', leaving it open for the informer to respond as he or she wishes.*[1]"

This has particular importance in Chance and Data when we ask students to think about answering questions by collecting data. For example, with activities from Key Understanding 1 that suggest that we model the posing of simple questions, care must be taken, on the one hand, to acknowledge and respect the different forms of questions that communities may find acceptable and, on the other hand, to help all students develop the ways of speaking used within mathematical discourse.

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[1] Cahill, R. (1999). *Solid English*. Perth: Education Department of Western Australia. Page 25. See also, Malcolm, I., Haig, Y., Konigsberg, P., Rochestcouste, J., Collard, G., Hill, A. and Cahill, R., (1999). *Two-Way English*. Perth: Education Department of Western Australia.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Question Box

Invite students to ask questions and decide which can be answered with data. Have them sort questions from their class question box into categories according to whether they can answer them:

- from their own experience
- from an information source such as a book or a website
- by producing some data.

Students place the questions onto a Venn diagram showing the three options, and add to it over time. Can some questions be answered in more than one way? How do you show that on your diagram? (See Key Understanding 3, link to Summarise and Represent Data, Key Understanding 4.)

### Why Not?

Ask students to say why they wouldn't use data to answer certain questions. For example: Why wouldn't it make sense for you to collect data to find out what day Christmas fell on this year? Do you have to go to different calendars to find out the answer to this question?

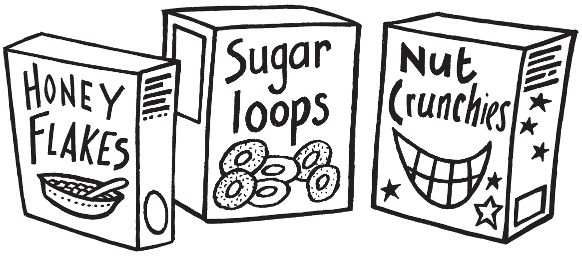
### Balloon Power

Encourage students to clarify their questions so they know what data is needed. For example, during a science lesson involving the construction of balloon-powered cars, students could examine a question such as 'Which car works best?' Ask: Would you be able to use data to answer this question as it stands? Why? Why not? Have students rework the question and come to a consensus about a question that can be answered using data. For example: Which car travels the longest distance? They then compare the new question to the original. Ask: Is that what you really wanted to know or is it something else? (See Key Understandings 2, 3 and 4.)

### Predictions

Ask students to formulate questions based on their hypotheses or predictions. For example, in the science lesson mentioned in the previous activity, students could predict factors that may affect how far the cars travel and then frame questions about those factors. Do long cars go further? Does the weight affect the car's performance? Do the cars that go further have large wheels?

### Is It True?




Have students reframe topical statements or hypotheses into questions, so that they can use data to find out if the statement is true. For example: Present a statement like 'Supermarkets place cereals with high sugar content on shelves at the eye level of young children'. Ask: How could we find out if the statement is true? What questions do we need to answer to help us find out if it is true? (See Sample Lesson 1, page 92.)

### Families

Have students define terms in a question so that data can be produced to answer the question. For example: Initiate a discussion about what is meant by 'family', perhaps comparing what restaurants include in their 'family meal deals'. What do restaurants consider to be a typical family? Ask students to consider who counts as family in their minds, in their home culture or in government statistics. You may need to take some children's family situations into consideration when talking about 'our family'. Talk about the idea of the 'nuclear family' and the 'extended family'. Ask students to describe their own definition of family and then, through a process of consensus, arrive at a definition of family to be used in their data collection.

### Producing Data

Ask students to decide if their questions can be answered by existing data or whether they need to produce the data themselves. For example: Should you carry out a survey of family size or go to census data on the Internet? Which data helps you answer your original question? How?

### Using Data

Invite students to frame questions from existing data or from data they have collected themselves. For example: Allow them to explore databases on the Internet as a stimulus to asking questions they find interesting. For example: Find the last census data and ask which professions are the most popular for women under 30 years old. Can you answer your questions directly from the data? Do you need to find other data to answer your questions?

### Mathematical Data

As the opportunity occurs, have students record their mathematical questions and decide which would require them to produce data. For example: What do we mean by 'polygon'? Is $3 \div 4$ the same as $4 \div 3$? Are sixes harder to get on a die than other numbers?

# SAMPLE LESSON 1

**Sample Learning Activity:** Later—'Is It True?', page 91

**Key Understanding 1:** We can answer some questions (and test some predictions) by using data.

## Motivation and Purpose

After considering some of the persuasive techniques used in advertising, my class of 11- and 12-year-olds was becoming more questioning of the media. So I wrote this statement on the board:

> Supermarkets place cereals with high sugar content on shelves at the eye level of young children.

I then asked: *How could we find out whether this statement is true?*

## Connection and Challenge

The students were keen to share their opinions with the class.

Antoinette: *We could keep a record of overweight children.*

Josh: *Or we could ask a dentist to ask the children with bad teeth what cereal they eat.*

There were other similar responses. It seemed many did not think carefully about what data would be useful to answer the question I had actually asked. With this in mind, I said: *Let's say a dentist gave us a list of cereals eaten by children with decayed teeth. How could we use this list to help us decide whether supermarkets do put the cereals with high sugar content on the shelves at the eye level of young children?*

Tony: *Well, I don't think we could. We still wouldn't know which shelves the cereals with high sugar content were on.*

Alister: *I know! We could catch them stacking the shelves on a surveillance camera.*

I responded: *But what would you actually be looking for?*

Elizabeth: *We'd be able to see what shelf they were putting the ones with high sugar on.*

Akila: *But we would need to know which cereals have the high sugar content so we could tell. We need to look on the packets.*

The students were beginning to isolate which data could help, so I prompted them to refine the question. *So if you find the sugar content of all the cereals, how will you know which cereals have a high sugar content? A word like 'high' can be tricky. How much sugar is 'high'?*

Rebecca: *Well, we could ask a dietitian about what is high sugar.*

Damien: *Or we could look on the packet and just pick the five highest ones.*

Satisfied that the students were beginning to grapple with this idea, I challenged them to look carefully at the initial statement to see if they would need to make other decisions.

Mark: *We don't know which shelves are at kids' eye level because some kids are tall and some are short.*

Josh: *And we don't know what counts as 'young'!*

***During this discussion and the following lesson, opportunities arose to talk to the students about whether it would be reasonable to generalise from their data. For example, if the students only produced data from one supermarket, would it be reasonable to generalise their findings to all supermarkets? Similarly, if they chose to measure the eye level of children and calculate an average, how many and which children would need to be measured? (See Key Understanding 5.)***

## Drawing Out the Mathematics

Through discussion, I drew from the students that to find out whether the initial statement was true, they would have to reframe it into questions that could be answered directly by data. I also drew out that more than one set of data might be needed. The class brainstormed a list of questions we needed to answer. These included:

- Which cereals have high sugar contents?
- Which shelves are at the eye level of young children?
- What will we call 'high'?
- What will we call 'young'?
- What is the eye level of young children?
- What shops will we call 'supermarkets'?

The focus of the next lesson was for the students to consider how they would produce the data to answer each of these questions. (See Key Understanding 2.)

## KEY UNDERSTANDING 2

***We can produce data by: counting or measuring things, asking groups of people, watching what happens, or reworking existing data.***

In developing this Key Understanding students should begin to learn that:

- it is the information we record about an object, event or experience that is our data
- we say that we 'collect data', but really we produce it
- there are a range of ways we produce the data that will help us to answer our questions
- the data does not tell us everything about the original objects or events or experiences
- we can only answer questions about the aspects of things that we have data on
- we should think ahead and try to imagine how we will use our data and how useful it will be.

### Counting or Measuring Things

Data production often involves counting or measuring things in fairly straightforward ways, and in the early years most data will be of this kind. As they progress, students should learn that when we collect frequency data (e.g. How many children like each type of book?), we may lose information that is difficult to retrieve later (e.g. Which children like each type of book?). They need to make conscious decisions about whether frequency information is sufficient or whether information about each case should be recorded. In addition, older students might begin to use simple rating scales.

### Asking Groups of People

Asking people to tell you what they've done, what they want to do, or what they think can be a useful way to get information. However, 'asking people' is no simple matter. Students need considerable opportunity to explore the effect on responses of wording questions in different ways, and of the effect of the type of response required (oral or written, yes/no, forced choices, simple information, open-ended). As their experience increases, students should be encouraged to anticipate what sorts of responses people might give.

## Watching What Happens

Recording our observations of things as they are happening or of the effect of things that have happened is another way we produce data. These may involve naturally occurring events (e.g. what students choose at the canteen) or involve an experiment (e.g. rearrange the food in the canteen on successive days and record what students choose). Students should begin to ask themselves what exactly it is that they will record.

## Reworking Existing Data

Students should learn that we do not always have to produce fresh data to answer a new question. We may be able to use data others have produced. We may also be able rework (reorganise) data we or others originally produced to answer other questions, and to use data on one thing to answer questions about another (e.g. using the number of mats laid out to find out how many students came to school today.)

## Progressing Through Key Understanding 2

Initially students offer suggestions about what objects they could collect or make to answer simple questions posed by the teacher. As students continue to progress they participate in group discussions about how to create data to answer specific questions. Next, they will suggest for themselves what data to collect to answer questions that make sense to them, and will try to clarify questions to make it clearer what data to collect. As students progress further, they will suggest indirect ways of getting data when direct data is not available. They will also attempt to reframe a simple survey question to make it less ambiguous and to make responses easier to interpret. Later, students are able to work in small groups to survey people, observe aspects of their environment, measure things, run experiments and generate data mathematically. They work autonomously but collaboratively in developing and trialling short sets of questions.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔✔

### Playground Survey

Display a picture map of the playground showing the outdoor equipment. Have students attach a 'self-action figure' (i.e. a picture of themselves) to the place or the piece of equipment they enjoy using the most. At mat time, ask students to count to see what activity most of them enjoy. Extend this by asking them to look for different information. Ask: What else can we find out? (e.g. There are more boys than girls on the slide.)

### Modelling

Model the process of deciding what data to collect by making suggestions about how to collect information to answer students' own questions. For example: Perhaps we could count how many children like different sorts of foods or perhaps we should watch what everyone eats for lunch. Which would best tell us which is our favourite food?

### Shoelaces

Ask students to decide what information they need to collect so that they can find someone to tie up their shoelaces when they come undone. Ask: Will knowing how many students can tie shoelaces help? Would a list of everyone's names help? (See Key Understanding 3; Link to Interpret Data, Key Understanding 1.)

### Scary Things

After reading a scary story, such as *In the Middle of the Night* (Graham, 1989), ask students what scares them the most. Ask: How could you find out what scares most students in this class, or in this school? Should everyone say, write or draw what scares them the most? (See Key Understanding 3 and Interpret Data, Key Understanding 3.)

### Birthdays

Have students write their birth month on a card and then find other students with the same birth month. Ask: Which month has the most birthdays? How can we find out? Should we count the groups, or would matching children one-to-one in the different months help? (See Summarise and Represent Data, Key Understanding 1.)

### Popular Toys

Invite students to decide how to collect information to answer questions such as: What is the most popular toy in Year 1? Draw out that asking everyone in the class is a sensible way, but not the only way. Ask: Would asking everyone give us the best idea? How else could you find out?

### Counting

Ask students to write a list of things they can count , e.g. how many of each coloured pencil they have. Then ask them to think of questions that could be answered by counting the things they have listed. Ask: Do you need to count the things in different ways to answer different questions? Compare the questions 'How many pencils do I have?' and 'How many pencils of each colour does our group have?'

### Grandpa's Breakfast

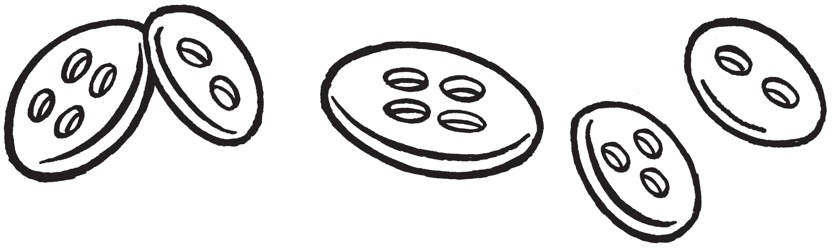
Extend 'Grandpa's Breakfast' (page 87) by asking: Where could you find the information to answer a question like 'What breakfast cereal has nuts in it?' By looking on the supermarket shelves, asking people in our class, looking on breakfast cereal packets?

### Out at Play

Ask students how they could decide how many children there are in another class when their classroom is empty. For example, they could count chairs, desks, bags, hats or lunch orders. Ask: How can we be sure that what we have counted will tell us how many students in that class are at school today?

### Buttons

Have students decide how they could find out whether most people wear buttons with two, three or four holes in them. Ask: Would asking everyone in the school help? Does wearing a school uniform make a difference to our result? Students could interview their parents and others in their community instead. (See Key Understanding 5.)



### Getting to School

When considering how students get to school in the mornings, ask: Would we get better information by observing others or by asking how they get to school?

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Tosses

Have students toss a fixed number of counters that have a different colour on each side, e.g. 12 red/blue counters. Ask them to decide how to record what is happening with each toss, in order to keep track of the colour combinations. Ask: What is it that we need to record? (See Key Understanding 3; Understand Chance, Key Understandings 4 and 6.)

### Which Method?

Have students compare methods of collecting data. For example: Which method should be used to decide whether there are more girls or boys in the school? Data could be collected by tallying students as they arrive at school, using the class roll, counting everyone at an assembly, or students could all pair up. Ask: Which method would be the simplest? The most helpful?

### Finding Information

Ask students to prepare one or two questions to collect data about their classmates, e.g. relating to their after-school activities and interests. Have students swap questions and complete them. Students then read the responses of others and reflect on the usefulness of their questions. Ask: Did your question give you the information you wanted? Was there any information you weren't expecting to get? Could you change your question to make sure you get the information you want?

### Popular Foods

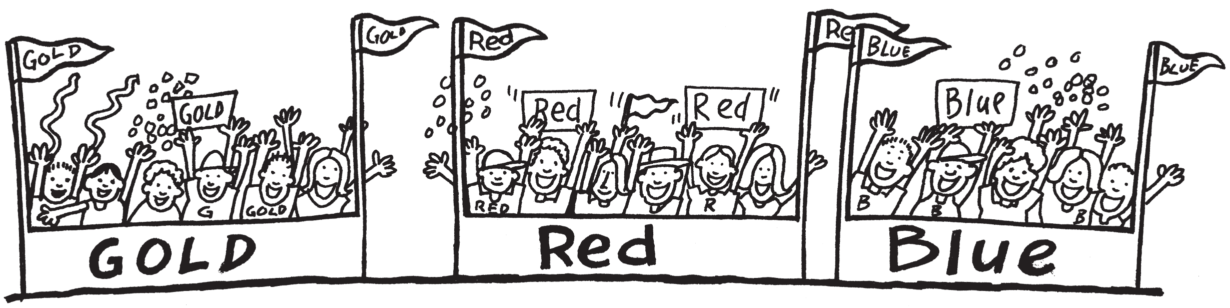
Have students collect data in different ways and compare results. For example: To find out which are the most popular foods at the canteen, one group might check sales for a week and another group might survey students. Ask: Did you find the same information? Which method best helped you answer your question?

### Paper Chase

Ask students to decide how they can find out how much paper is used in the school for photocopying. Ask: Could you find out by using information that is already in the school?

## Sports Day

Invite students to decide how to collect data during a sports carnival. Have them brainstorm different types of data they could collect to write an article for the school magazine. Ask: What would people like to know about our carnival? How could we collect this information? Students may observe the house/faction bays at various times in the day to decide which is cheering the most, or they may decide to use the information recorded for the long jump to compare the jumps of different age groups. Ask: Did your method of collecting data help you to write your article? Should you have used a different method to collect your information? Why?




## Speeding Up

Have students rework data produced to answer one question, in order to answer other questions. For example, children collect data to answer the question 'What are the fastest times for the different track and field events in the last few Olympics?' They then use this data to answer other questions such as 'Are the athletes really getting faster?'

## Recording Data

Have students decide how to record information so that nothing is lost in the process. For example: Are students from the same family always in the same house/faction? Ask: Can you record this so that you could find out which family is in which faction? Can you record it so that you know the names of the children in each family? If you record it as a tally, what information would be lost?

## Question Box

Encourage students to consider a range of ways to produce data that will help them answer a question. Have pairs of students take a question of interest from a class question box. Ask: Is it possible to answer the question by counting or measuring something, asking groups of people, watching what happens or by reworking some existing data? Which of these ways would you choose to answer your question? Why? (See Key Understanding 1.)

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Counting on Frank

After reading *Counting on Frank* (Clement, 1990), ask students what question the boy in the story might have been asking himself for each calculation. Have them then decide what data needs to be collected to answer each of the questions. (See Key Understanding 1, 'Middle'.)

### Cereal Survey (1)

Have students each prepare a two- or three-question survey to collect data from their classmates about a particular new cereal. For example: How many times have you eaten the cereal? Did you enjoy it? What did you enjoy about it? When did you feel hungry again after eating it? Would you recommend your parents buy it again? Do you think it is good value for money?

### Cereal Survey (2)

Ask students to test and revise questions. For example: Before using questions from the previous activity in a real survey, have them swap questions, complete them and return them to the owner. Students then read the responses and reflect on the usefulness of their questions. Ask: Did your question give you the information you wanted? Was there any information you weren't expecting to get? Could you change your question to make sure you get the information you want?

### Cereal Survey (3)

Have students decide whether the answers to the questions in the previous activity could be recorded using numbers or words. Ask: How does this affect how you will record your data?

### Anticipating Answers

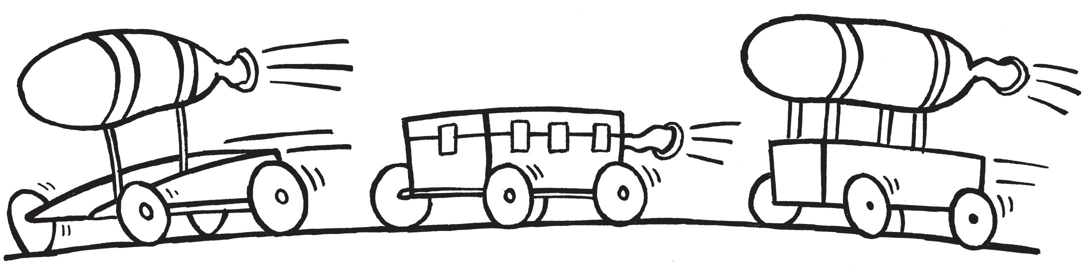
Ask students to anticipate what people might say in response to a question before deciding how to collect and record the data. For example, they anticipate responses to an open question like 'What did you enjoy about … (a certain book, movie, new cereal)?' Ask students to consider asking people to rate their enjoyment on a 1–10 scale. Ask: Which data gives the best indication of people's enjoyment? Why? When would it make sense to use a rating scale and when wouldn't it?

### What Kind of Response?

Invite students to decide between fixed choices or open-ended responses when planning a short series of questions. They consider whether the responses to questions should be in the form of a tick, a word, a sentence or a paragraph, by discussing the benefits and risks of each. Draw out that while collecting data in categories may close some options, it may open up possibilities that students had not thought of before. The responses from open-ended questions may be difficult to sort but give broader information.

### Balloon Power

Encourage students to think ahead to what data will answer their question. For example, have them brainstorm data that might help them find out which balloon-powered car works the best. Ask: Would frequency data of the number of cars travelling certain distances be enough to answer the question? What information would you record prior to or during the trial? Is there other information that you could record during the trial that might help answer the question? (See Key Understandings 1, 3 and 4.)



### Make a Spinner

Have students collect data to test predictions. Ask them to construct a spinner that they think is most likely to stop on red, least likely to stop on green and have the same chance of stopping on yellow and blue. After they have designed their spinners, ask: How do you know it works? Students decide what data they will need to collect to test the spinners they designed. (Link to Understand Chance, Key Understanding 2.)

### Pocket Money

Ask students to consider how the way a question is asked can bias responses. For example, have them write, trial and revise two or three questions to find out what parents think about the idea that children should only get pocket money if they do special jobs at home. Compare responses to an open-ended question, a multiple-choice question, a yes/no question, and a statement with a rating scale. Ask: How does each method affect the data you get?

# KEY UNDERSTANDING 3

## *Organising data in different ways may tell us different things.*

In developing this Key Understanding students should learn that:

- classification underlies the organisation of data
- how we classify depends on the questions we want to answer
- data organised in different ways may tell us different things
- data organised in a particular way may expose some things and mask others
- new questions may require a reorganisation of data
- reorganising data often suggests new questions for investigation.

A central part of designing data production is thinking carefully about how data is organised during the collection phase, and how it might ultimately be further organised and reorganised for analysis. The process of producing data is usually based on classifications (e.g. do we just record 'bird' or do we record different types of birds?) and it is important to think about this ahead of time. Also, data is often recorded in an 'organised' way (e.g. in a tally sheet) and the form of organisation used can determine how useful the data is. Planning ahead in this way is not an approach students commonly use. Possibly the best way for them to learn to do so is from seeing what goes wrong when they don't. Teachers can also model the process, not by making all the decisions for the students, but by exposing their own thinking processes and providing scaffolding to enable the children to plan for themselves.

In the lower primary years, students will classify and sequence in straightforward ways to answer simple questions. Initially, they classify and sequence actual objects or pictures of objects, rather than data. Later their classifications broaden to include data represented in various forms such as paper strips, words and numbers. Even relatively simple classifications can be ambiguous (e.g. Is it blue or green?) and students need experience with the kinds of dilemmas people face when making decisions about whether an item belongs to one category or another. If the purpose of classifying is clear and makes sense to them, students are more likely to recognise ambiguities in their classifications and try to clarify or improve on their descriptions of categories.

Students should be expected to make suggestions about how to classify and sequence their data in order to answer particular questions. Over time they should explore the effect of different classifications on what they can or cannot learn about the things they are investigating. Thus, in investigating their favourite sports, one way of organising their data might suggest that students generally prefer ball games over non-ball games. However, another way might suggest it isn't ball games students like so much—rather that they prefer team to individual sports. Similarly, one way of classifying favourite books might enable students to decide whether children like books that have pictures better than those that do not. Another way of organising the books might mask this distinction. Older students should develop strategies for improving their efficiency in sequencing and classifying larger sets of data, and represent two-way classifications in diagrams and tables.

## Progressing Through Key Understanding 3

Initially students can apply familiar and unambiguous criteria to classify and sequence data consistently. As students continue to progress, they make suggestions about how to classify their data to answer straightforward questions, and are beginning to understand that the same data might be reorganised to answer different questions.

Next they are consciously aware that different classifications may be necessary to answer different questions, and can suggest how to improve a classification strategy to better suit the purpose.

As students progress further they can plan class intervals as a way of organising and classifying their measures. For example, they may find the mass of various rocks and group the data from 0 to 250 g, more than 250 g, up to 500 g, and then over 500 g. They can also organise data in databases.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔

### Sorting

Ask students to sort a collection of things such as shells, leaves, seed pods or toys, and make general statements about how they are the same/different.

### Building

Have students sort objects into those that are useful for building and those that aren't. Discuss their reasons for grouping them, then construct a house and say if they change their mind about the grouping. (See ***First Steps in Mathematics: Space***, Reason Geometrically, Key Understanding 1.)

### Shoeboxes

Invite each student to keep ten different things in a shoebox to sort. Ask students to sort them according to different criteria on different days. Use their suggestions for groupings and include a group of things that 'are not'. (See ***First Steps in Mathematics: Space***, Reason Geometrically, Key Understanding 1.)

### Labelling Groups

Have students collect a container full of different items from inside/outside their classroom and sort them using their own groupings. Students make labels for groupings, but hide them. Other students then guess what labels they have used for their groups. When the label suggested is different, ask: Could both labels be used for the group?

### Different Groups

After the previous activity, have students sort the same collection into different groups. Ask: Is it possible to put them into two groups instead of four (or four instead of two)? What would you call each group?

### Shoelaces

Ask students to decide what information to collect so they can find someone to tie up their shoelaces. Ask: When you ask people, how can we keep track of who can and who can't tie up their shoelaces? Would writing down the names of the students help? Which names should we write down? (See Key Understanding 2; Link to Interpret Data, Key Understanding 1.)

### Transport Groups

Have students create a chart using pictures of different sorts of transport. Ask them to decide on their own categories and then compare their groupings with those of others. Ask: Why do the groups show different things? Is one way of grouping better than another?

### Once Upon a Time

Invite students to find more efficient ways to organise their data. For example, after reading *Once Upon a Time* (Prater, 1998), have them work out how many different characters are in the book. Ask: How could we organise the data so that we don't miss any of them, or count them twice? Encourage students to list the names of the different stories included, tally the characters for each and then find the total. (See Key Understanding 4.)

### Language Data

Have students organise their data into their own categories and then compare how others have organised their information. For example, after they have recorded the different languages spoken in the class, ask: How could we organise the information to make it easier to read? Draw out that lists and tables help us organise our data. (Link to Summarise and Represent Data, Key Understanding 4.)

### What Is Work?

Ask students to clarify categories to sort objects. When investigating topics in Society and Environment such as 'working in the home', ask students to define what can be included in their categories. Ask: What would we include as 'work'? Is cooking tea 'work'? Is helping with cooking tea 'work'?

### Where Do You Fit?

Invite students to decide which of two categories they belong to. Place labels around the room so that all students fit into one category or the other (e.g. boys/girls, wearing sneakers/not wearing sneakers, brought their lunch with them/didn't bring lunch). Have students decide where to stand. Ask: What made you decide to stand on that side of the room? What do we know about Claire if she is standing over here?

### Favourite Vegetable

Ask students to draw a picture of their favourite vegetable on a small square of paper and then arrange all the pictures into groups according to the type of vegetable. Have students count the groups to work out which is the most popular. Ask: What would happen if we placed our vegetables into groups according to their colour? Students then count the new groups and say which colour of vegetable is the most popular. Ask: Why do we get different results from the way that we group our pictures?

### Scary Things

Extend 'Scary Things' (page 96). Ask students to draw a picture of the thing that scares them the most and sort the pictures into categories that they decide on. Students need to work out how to deal with pictures that don't seem to go into their existing categories, or belong to two categories. (See Sample Lesson 2, page 112.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Where Do You Fit?

Ask one student to decide on two categories (e.g. blue eyes, not blue eyes) and start to put other students into these two groups. Invite the others to work out what the categories are by looking at what the students in the groups have in common. Extend this to include attributes that are more ambiguous, e.g. light brown and dark brown hair.

### Recording Information

After the previous activity, help students to record the information in a table. Ask: What were our two categories? (Write this on the board.) Which students belong in each category? (Record the names under each heading.) Draw out that organising information in a table helps us to see at a glance who belongs in each group after they have physically moved away.

### Food Groups

When creating a table of different food types, ask students to clarify the categories that have been used. Have them add items to the table to show what foods they have eaten. Ask: What sorts of foods would we add to the fruit and vegetable group? What foods would we add to the dairy group? Where would we put things that are made up of foods from different groups, such as chocolate cake?


### Classifying Objects

Have students classify objects using ambiguous classifications. For example, a collection of leaves or rocks could be sorted according to size and/or colour. Ask: Are there more large ones or small ones? Are there more brown ones than red ones? When students begin to struggle with placement, ask: What is the problem? How could we decide where to put that item? How can we clarify what we mean by large and small?

### Categories

Have students decide on categories before asking their questions or observing the situation, and then decide how useful their categories were. For example, prior to a class excursion to the zoo, students might have chosen the categories 'hair' and 'fur' to group animals. During the excursion ask: Were there some animals you found difficult to put into these groups? How could you change the groups to make it easier? Do these groups help us to answer the question that we now have?

### Birthdays

After students have answered one question with data they have collected, ask another question that requires them to reorganise their information. For example, after they have worked out which months students have birthdays in, ask which season most birthdays are in. Ask: What do we have to do with the data?

### Tosses

Invite students to choose a fixed number of counters with a different colour on each side, say eight red/blue, and list all the possible outcomes when the counters are tossed together (eight blue; seven red and one blue; and so on). Ask them to decide how they can record the information to show how many times each pair of colours come up. Students can record as they go, then compare the different recording methods and say which are easier to use to record, and which are easier to read. Draw out that lists and tables are useful in recording information in an organised way. Help students to decide which table would be the easiest to use to record the information and repeat the activity using this table. (See Key Understanding 2; Understand Chance, Key Understandings 4 and 6.)

### Getting to School

Have students decide how to create a table to record information on how students get to school. Ask: Would you list all of the modes of transport and tally under each category as you ask? Students can then say whether they were able to record all information on the table, or whether it needs to be modified. Ask: How else could you have created the table?

### Favourite Sport

Ask students to organise information and then try different categories to see if this changes the result. For example, ask them to write their favourite sport onto a sticky label and then decide how to group the labels to answer the question 'What is our favourite sport?' Ask: What does this organisation of the information show? How else could we organise this information? Have them rearrange the sticky labels and then decide whether the new organisation shows something different. Ask: Which organisation best helps us to answer our question?

# Middle ✔✔✔

### Venn Diagrams (1)

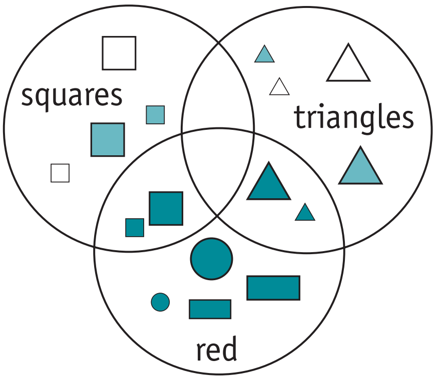
Have students decide which category they belong to and place themselves into a Venn diagram created with rope or tape. Start with two separate circles and ask students to stand in one circle if they play tennis and the other if they play soccer. Ask: How can we show that some people play both? Move the circles so that they overlap. Students then stand in the appropriate place. Ask: Where do people stand if they play neither? (Outside both circles.)

### Venn Diagrams (2)

Extend the previous activity to include a third group such as tee-ball. Start with another separate circle. Ask: How could you show that some people play both tee-ball and tennis? How can we show that some people play three different sports? Move the circle so that it overlaps the first two and have students move into the appropriate place.

### Venn Diagrams (3)

Invite students to identify discrete categories by using Venn diagrams. For example, ask them to sort attribute blocks into three circles showing squares, triangles and red shapes. Ask: Why aren't there any shapes in the overlap between squares and triangles?



# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Sorting Cars

Ask students to sort pictures of similar objects and describe the spatial likenesses and differences between them. For example, what is the same/different about the shape of a:

- Volkswagen and Mercedes
- mountain bike and BMX bike
- kitchen chair and lounge chair
- kangaroo and wallaby
- running shoes and basketball shoes?

Have students give a partner some of the pictures and have them sort into their categories. Were they able to use your categories? Are some of the categories ambiguous? How can you clarify your categories? (See ***First Steps in Mathematics: Space***, Reason Geometrically, Key Understandings 1 and 3.)

### Food Groups (1)

Ask students to decide on their own food groups and define what characteristics each group will have in common. Have them record everything they eat and drink over a day and sort their list into their groups as a way of categorising what they ate. Ask students to define their categories and see if a partner can use these to sort their own list of food. Ask: Is it possible to sort all the foods into these groups, or do you need to define another group? Ask: How are these groups similar to or different from the standard food groups? Which way of grouping is the most helpful in designing a healthy eating plan?

### Food Groups (2)

Extend the previous activity and have students compare how they and their partner presented their information. Ask: How did you show what your categories are? How did you show what is in each category? Students compare how they presented their information in a table to other ways of presentation.

### Sorting Students

Extend 'Venn Diagrams' (page 108) to topics that include more categories. For example: Students decide on three categories of pet ownership and show which category they belong to by standing in a Venn diagram created with rope or tape. They overlap circles to show that some people own more than one type of pet. Ask: Where do people stand if they don't own a pet or do not own a pet that is in one of the categories? Have students rename the categories to include more children.

## Later ✔✔✔

### Question Box

Have students sort questions from their class question box into categories according to whether they can answer them:

- from their own experience:
- from an information source such as a book or a website
- by producing some data.

Ask students to place the carded questions onto a Venn diagram showing the three options and add to it over time. Ask: Can some questions be answered in more than one way? How do you show that on your diagram? (See Key Understanding 1.)

### Favourite Sport

Invite students to suggest how to change a classification to answer different questions. For example, students write their favourite sport onto a sticky label and together organise the labels to see which event is liked most by boys and which is liked most by girls. Ask: Does this tell you what event is most liked by all students? How would you reorganise the labels to find out? (See Sample Lesson 3, page 114.)

### Favourite Food

After using data to answer a particular question (such as 'What is our favourite food?'), ask students to decide whether their data could be used to answer a different question. For example: Do we like to eat healthy foods? Do we prefer burgers or chicken? Is price an issue? Do students in our class like more expensive food? Ask: Would the same organisation of our data answer this new question? How could we organise our information to answer our new question?

### Pocket Money (1)

Ask students to organise their data in a one-way table electronically (e.g. a table with 'sort' options or a spreadsheet). For example:

| **Amount of pocket money children receive each week** | | |
|---|---|---|
| **Amount ($)** | **Gender** | **Work/Don't work for it** |
| 5 | G | DW |
| 7.50 | G | W |
| 5 | G | DW |
| 10 | B | W |
| 5 | B | DW |

Can you see how much money children receive each week from this table? Can you see any interesting trends?

### Pocket Money (2)

Extend the previous activity to see that sorting in different ways can expose some things and mask others. Students use the 'sort' option to sort by the different columns and then say which way of sorting the information best helps them to answer their question. Ask: Does one way of sorting hide the 'interesting' information? When might you use this way of sorting?

### Balloon Power

Have students structure and record data electronically (e.g. a table with a 'sort' option or a spreadsheet), then sort to answer specific questions. For example, they could record data from the trials of their balloon-powered cars. They then sort the data in different ways to answer questions such as: Is there a relationship between the distance travelled and the weight of the car? Do the cars with larger wheels travel further than the cars with smaller wheels? (See Key Understandings 1, 2 and 4.)

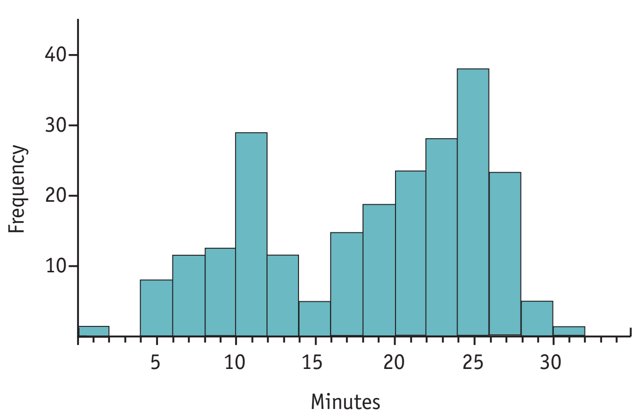
### Two-way Tables

Invite students to decide how to set up a table to investigate relationships in simple data. For example: Which is most popular—white or wholemeal bread with Vegemite or peanut butter? Ask: How could you record the information in a table as you gather it? Could you use labels on both the columns and the rows? Does the table help us to see at a glance which is most popular?

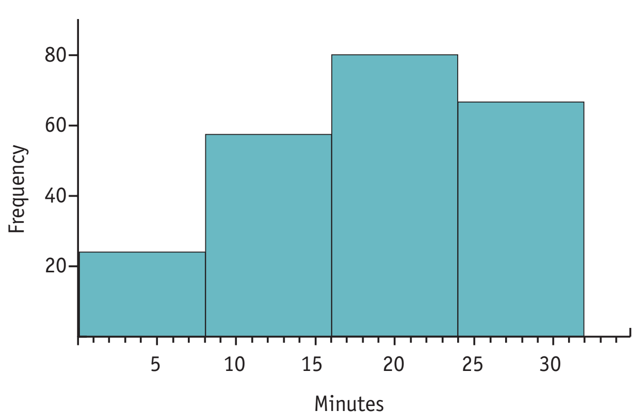
### Graphing

Ask students to use graphing software to see how grouping measurement data into different-sized intervals may expose some things and mask others. For example: Examine graphs such as those below and say how the different intervals affect what you can see in the graph.

**Length of time it takes to get to school**




**Length of time it takes to get to school**



# SAMPLE LESSON 2

**Sample Learning Activity:** Beginning—'Scary Things', page 105

**Key Understanding 3:** Organising data in different ways may tell us different things.

## Motivation and Teacher's Purpose

A group of Year 2 students were in the classroom reading corner.

Jasmine: *I love this book. It's about monsters and monsters are scary.*

Harry: *Monsters aren't scary.*

Jessiah: *Yes they are!*

I recognised an opportunity for the students to produce and organise some data!

*I noticed that many of the students' pictures depicted more than one idea, such as a monster in a dark room. Production of authentic data may often mean that students are dealing with 'messy' data that does not fall easily into neat and obvious categories. However, students need experience in making decisions about organising this type of data if they are to achieve the outcome.*

## Action

I read the class the first few pages of a story about a little dog that imagined all sorts of scary things. I then presented them with a question: Is there something that scares most of us?

The students each drew a picture of what scared him or her most and took turns to tell the group about their pictures as they stuck them haphazardly on the board. I led a discussion in which the students decided to sort their pictures and count the number in each group to answer the question. Soon they began to argue about whether monsters and ghosts belonged in the same group, eventually agreeing that ghosts were white and monsters were any other colour. They continued to sort using their chosen categories of: monsters, ghosts, sharks, crocodiles, spiders, bad dreams, lightning, and the dark.

## Drawing Out the Mathematics

I drew my students' attention to a picture of ghosts looming in a dark room: Where will we put this picture?

Michael: *That one is ghosts so it goes in the ghost group.*

Elizabeth: *But it should be in the dark group!*

Me: *That picture could go in two groups. How can we decide where to put it?*

Justin: *I know. I think we should look at what's most in the picture. There's only a little bit of dark and lots of ghosts so it's a ghost picture.*

The other students consented and continued to classify pictures until they came to a picture of a monster with the word 'dream' written underneath.

Jacob: *It's a dream picture because it's a dream!*

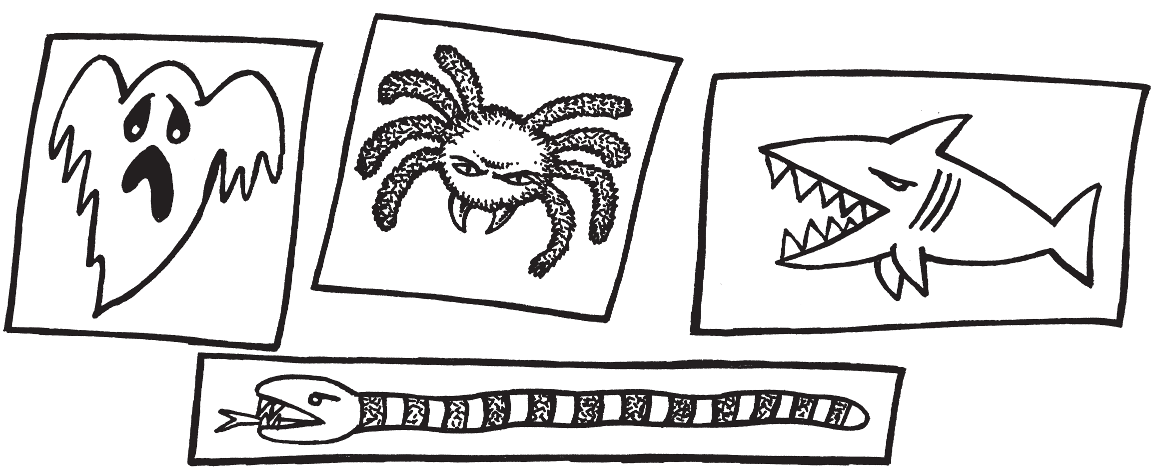
Maggie: *No, it's a monster picture. It's a dream monster.*

Me: *We will need to choose which group to put this picture into.*

Jacob: *But which is the right group?*

Me: *There isn't a 'right' group. We have to make up our minds about what groups we want so we can sort the other pictures in the same way.*

They sorted the remaining pictures and counted the numbers of pictures in each group. They concluded that bad dreams scared most of the children.



## Reflection

I was satisfied that my students had participated in making decisions to organise some 'messy' data. Next lesson I planned to draw their attention to the possible 'different answers' they may have reached if they had made other decisions during the organisation process.

# SAMPLE LESSON 3

**Sample Learning Activity:** Later—'Favourite Sport', page 110

**Key Understanding 3:** Organising data in different ways may tell us different things.

## Motivation and Teacher's Purpose

Some Year 4 students were discussing which events they had liked and not liked at the school's recent athletics carnival. At my suggestion they were keen to further investigate their preferences. I anticipated that the type of data they would collect would lend itself to a discussion about organising data in different ways.

## Connection and Challenge

*In this case the data could have been first sorted according to event, and then according to gender.*

The students came to a consensus about the question they wanted to investigate: Did Year 4 boys and girls both like the same event best? They decided that each student in their class and in the Year 4 class next door, should write their name and the event they liked best on a sticky label. The labels were then stuck on the board and sorted, firstly into boys and girls, and then into events. The number in each group was recorded. The students saw that the flag relay was liked best by more boys than any other event and that pass ball was liked best by more girls than any other event. The Year 4 boys and girls did not like the same event best.

| | long jump | high jump | pass ball | running | flag relay |
|---|---|---|---|---|---|
| boys | 5 | 5 | 2 | 8 | 10 |
| girls | 7 | 4 | 9 | 6 | 5 |

## Drawing Out the Mathematics

Me: *So, if we were going to do just one of these activities during our sports lesson on Friday, how could we use this data to help us decide?*

Flavio: *We'd do flag relays because that has got the biggest number.*

Josie: *But that's what the boys like. The girls liked pass ball best and there's only five who voted for flag relays.*

Me: *We are trying to decide on just one activity for everybody to play. We might have to think about grouping our data in a different way.*

Sam: *Well, it doesn't matter if they are boys or girls—we all have to play. You have to look at the big groups. Like you have to add the boys and girls up.*

I invited Sam to draw on the board to help him explain his idea to the class.

Me: *Sam has reorganised our data into new groups so we can see how many people, boys and girls together, like each event best. What event is liked best by most Year 4 students?*

***I took the opportunity for students to consider that different classifications may affect what they can and cannot find out about things they are investigating.***

| long jump | high jump | pass ball | running | flag relay |
|---|---|---|---|---|
| 5 | 5 | 2 | 8 | 10 |
| 7 | 4 | 9 | 6 | 5 |
| 12 | 9 | 11 | 14 | 15 |

George: *Flag relay, because that's got the biggest number.*

Fatima: *But that's still not fair. The girls like pass ball best.*

Me: *When our data are all together we can't see the difference between boys and girls. When we think about just the girls, pass ball is liked best by the greatest number. When we think about just the boys, the flag relay is liked best by the greatest number. But when we think about all Year 4 students, the flag relay is liked best by the greatest number.*

***The data could have been first sorted according to event and then according to class.***

Diedre: *But when we do sport on Fridays it's just our class. The other class isn't there.*

Me: *I wonder if this would make a difference to our answer? How could we group our data to help us find out?*

*Using sticky labels on the whiteboard enabled the students to physically reorganise the same set of data into new categories.*

After some discussion they decided to group the data into classes and then group the events within each class. They then re-sorted the data to find out if both Year 4 classes liked the same event best. The students were quite surprised to see that this reorganisation of data showed that neither pass ball nor flag relays was liked best by the greatest number of students in their class, nor in the other class. In fact, long jump was liked by the greatest number in their class and running races were liked best by the greatest number in the other class.

| | long jump | high jump | pass ball | running | flag relay |
|---|---|---|---|---|---|
| our class | 9 | 3 | 7 | 4 | 7 |
| other class | 3 | 6 | 4 | 10 | 8 |

*If the data had not lent itself to demonstrating these ideas, I could have focused on recording this type of information in two-way tables and then used another data set on another occasion to explore the effects of organising data in different ways.*

I was satisfied that my students had seen that organising data in different ways may tell them different things, and that how they organised their data depended on what they were trying to find out. They had also had an opportunity to see how organising data in a particular way may expose or mask some things.

*Students are able to make suggestions about how to classify data to answer straightforward questions, and are beginning to understand that the same data may be reorganised to answer different questions.*

## Extension

I decided to work with my students to show how the same information could be recorded and sorted in a table or spreadsheet on a computer. They went on to collect information about the best-liked events from the rest of the students in the school, and put all of their data into a spreadsheet program. They could see how using the computer to help organise their data made reorganising categories much easier than trying to manipulate their original data sheets.

### *Did You Know?*

Data is the record of our observations. To investigate how a snail travels across the garden, we can record different types of information about its journey:

- quantitative (its trail was 2.3 metres long) or qualitative (its trail was transparent and gooey)
- subjective (it was travelling really slowly) or objective (it is under that table).

We can record observations of the snail itself as it moves (in real time or on film) or reconstruct its path by observing the snail trail it left or the damage to plants along its route, or ask several people to remember and tell us about its path. Thus we could produce many different kinds of data from and about the one phenomenon. The data we choose to record depends on what question we want to answer.

# KEY UNDERSTANDING 4

## *We should make our data as accurate and consistent as possible.*

An important but subtle aspect of this Key Understanding is that we work with data itself rather than the original things and we cannot expect to be able to 'fill in the gaps' by adding personal knowledge to our data or recreating the circumstances that produced it. Students need to understand that data has to 'stand alone' and be good enough to represent features of objects, events and experiences that are no longer there. They need opportunities to make decisions individually and collaboratively, and note what goes wrong when they don't plan their data collection well so that, with the help of their teacher, they can improve their techniques.

We usually judge whether data is 'good enough' by two criteria—its accuracy and its consistency. Within statistics these are referred to as validity and reliability, respectively, although students do not need to use these terms.

### Accuracy

Being accurate (or valid) is about making sure that we are getting a 'true' measure or indicator of the thing we are interested in. When determining pet popularity by counting pets owned, we need to decide whether an aquarium of fish counts as one or whether each fish counts as one. Both numbers are 'correct' but the issue is which is the better indicator of popularity of fish over dogs and other pets. When asking for people's opinion we need to think carefully about how to ask the question so that we do not bias their responses. To test a prediction that the area of one's hand print is about one-fortieth of one's total body area (as used in burns wards), we need to know whether it is palm only or fingers as well. The production of accurate data links closely with Key Understanding 1, since we need to ensure that the way we reframe questions and define terms remains 'true' to the original question.

## Consistency

Being consistent (or reliable) is about making sure that we do not introduce chance variation into our data through erratic data collection processes. We want to feel confident that the same information would be recorded on different occasions or by different observers. Inconsistencies in data occur when we:

- are careless in making or recording our observations (e.g. not counting some of the people arriving, entering tally marks in the wrong column)
- do not do it the same way each time (e.g. sometimes leaving our shoes on when measuring height and sometimes not, sometimes including aqua as blue and sometimes as green)
- do not make sure that we are all doing it the same way (e.g. some measuring arm length from armpit to wrist and others from shoulder to wrist).

Students should be helped to consciously plan approaches to data collection that minimise these sources of error and variation.

## Progressing Through Key Understanding 4

Initially, as students are able to attend to accuracy and consistency, when prompted, they will clarify what to record in each category, e.g. whether an aquarium of six fish should be counted as six or as one in the fish category. They are beginning to be careful in their data collection, e.g. in making tallies they try not to miss any out.

As students progress further, they recognise the need to measure what you think you are measuring, e.g. they will think about how they phrase a survey question. They take care with their data collection but may not anticipate difficulties or plan ahead. Next, students will anticipate problems, plan ahead and do test runs to ensure that their measurements or frequency counts are accurate and consistent.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Sorting Toys

Have students sort a collection of toys into two groups: 'inside' toys and 'outside' toys. On a different day, ask them to sort the same set of toys again. Ask: How can we make sure that we always put the same toys into the same groups?

### Once Upon a Time

Invite students to find ways to organise their counting so they don't recount, or miss anything. For example, after reading *Once Upon a Time* (Prater, 1998), have them count to say how many different characters are in the book. Ask: How could we organise our counting so that we don't miss any of them, or count them twice? Encourage students to list the names of the different stories included, tally the characters for each and then find the total. Ask: Are there other ways that we could check that we have counted correctly? (See Key Understanding 3; Link to ***First Steps in Mathematics: Number***, Understand Whole and Decimal Numbers, Key Understanding 1.)

### Long Jump

Ask students to use a piece of paper tape to record how far they can jump, then say whether their jump is possibly the longest in the class. Ask: How can we make sure that each piece of tape is measuring all of a jump, and no more? Students share and compare where they placed the tape to mark the start, and where they tore the tape off to mark the end of their jumps. Help them come to a consensus about how to get an accurate measure.

### Favourite Colours

Encourage students to consider situations where data is collected inconsistently to influence the outcome. For example: Let the students know that your favourite colour is, say, blue and ask them to vote on which colour to use to make a sign for the classroom door. Count several blue votes more than once, and do not count some of the other votes. When students complain the vote isn't fair, ask: Why do you think it isn't fair? What would make it more fair?

### Favourite Vegetables

Have students collect data to find their favourite vegetable. Before they start, clarify what data is going to be collected. For example, ask: Are we going to give one favourite vegetable each, or several?

## Bingo

As students play Bingo, have them record the letters or numbers that come up on their sheet. Ask: How can we make sure that the person who says 'Bingo' first has correctly marked their card? Would it help to keep a record of all of the numbers or letters that come up as they happen?

## Foot Size

Have students each measure the size of their foot then compile the data and compare their results. Ask: Have we all measured the same part of our foot? Which part of our foot could we measure? If we were to compare the size of Year 2's feet with Year 7's feet, which part of the feet should we all measure?

## Language Data

After students have decided on categories to use to collect their information, clarify what they might put into each group. For example, when collecting data on the languages that are spoken in the class, ask: Where will you put someone who can speak both Chinese and Malay?

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Measuring Chickens

After students collect measurement data, have them check they have measured the same thing. For example, when measuring the growth of the class chickens using blocks, ask them to show how they measured a chicken. Ask: Did you all measure in the same way? Do you think it matters? Why? Draw out that to find out how much the chickens have grown, we need to get a 'true' measure of the chickens' height and we need to all measure in the same way.

### Making Hats

Have students collect measurement data and then consider why it matters whether they have measured correctly and all in the same way. For example: They measure the distance around the heads of Year 1 children to make hats. After using this information to make the hats, ask them why some hats did not fit.

### How Tall?

After students collect measurement data, have them review the numbers and see if they all seem reasonable. For example: Have students measure the heights of everyone in the class to find out if Year 3s really are taller than Year 1s. Ask: Is Jodie really taller than Zak? Does it matter if we have our shoes on or off? Does it matter how each person stands, or how we find the top of a person's head? Invite students to suggest how to make their measurements more accurate, and then re-measure both groups and compare the results with those obtained the first time.

### Car Colours

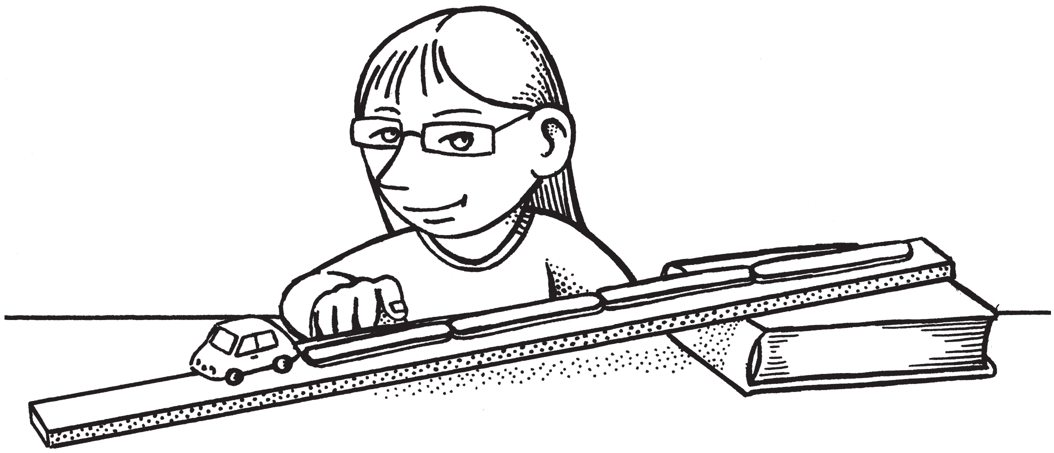
Have students make decisions about ambiguous categories. For example: Ask students to collect data on car colours, then wait a few days and ask them to collect more data on car colours. When ambiguities arise, ask: Who can remember what we did last time? Did we call 'silver' grey or blue? How could you have described the categories so you all sort the colours in the same way? Would more categories help avoid having to make difficult decisions?

### Different Methods

Invite students to compare different counting and recording methods across different situations. For example: They could use tallies, electronic counters, the constant function on a calculator and counting to say how many students were wearing yellow on 'yellow day', or to keep track of how many laps around the basketball court a student runs. Ask them which methods are most reliable (that is, consistent) for each situation.

### Model Cars

Encourage students to refine their measuring technique of an object or event prior to collecting data to answer a specific question. For example: Have individual students use pop sticks to measure and record the distance travelled by their model cars down an inclined plane. Ask: How is it that we can get different measurements for the same distance? How should we carry out our measurements so that we all get the same answer each time for the same distance? (See Key Understanding 1.)



### Keeping Track

After students have recorded data to answer a question, ask them to reflect on how easy it was to keep track of their data. For example: Have a group of students go around to each class to find out how many students from each faction/house are at school that day. Ask: What could you have done to make it easier to record your data and check that you had the correct amount? Would organising it into a table have helped? How?

### Checking for Consistency

Before collecting data, have students brainstorm ways of checking that their data will be consistent, e.g. working in pairs and cross-checking their results. Have them try out their suggestions and reflect on their usefulness. Ask: Did you both collect the same data? If there were differences, why do you think they occurred?

### More Scary Things

Have students collect and organise data about themselves in response to a question like 'What scared you the most when you were younger?' Students then choose how to collect comparative data from others, and reflect on the problems with accuracy that occur when they don't carefully plan and agree on the question and collection methods. (See Sample Lesson 4, page 126.)

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### What Are You Measuring?

When students take measurements to use as data, ask them to consider how to make their measurements accurate. For example: If they want to measure the length of their pets, do they include the tail; do they measure in a straight line from nose to tail; or do they allow the tape to follow the curves on the head and back? Which would be a better indicator of the length? Are you measuring what you think you are measuring? How do you know?

### Growth Experiments

Invite students to consider how to ensure measurements are consistent when recording data. For example: During growth experiments of living things, like the height of wheat plants, or the length and mass of baby guinea pigs, ask students to write instructions for taking the measurements so that different students at different times will take the measurements in the same way. Ask: How could our data be affected if we don't write clear instructions?

### Growth Patterns

Arrange for students to visit the pre-school or Year 1 classroom each month and take careful height measurements of each student to examine the growth pattern of young children. Explain that the overall growth will not be more than a few centimetres. Ask: So what do you need to think about to ensure your measurements are accurate and consistent? (For example, a standard height measuring instrument, ensure not wearing shoes, stand the same way each time, careful reading of the scale.)

### How Do We Measure?

When students are collecting measurement data, ask them to describe how they measured. Why does it matter if:

- we all use the stopwatch in different ways to check how long it takes people to walk a distance?
- when weighing the seeds they collected, some people weighed the container as well?
- when measuring to see how much children have grown, you leave shoes on one time and take them off another time?

Encourage students to say why it is important to measure correctly and all measure in the same way, and to predict what effect this would have on the conclusions they can draw from their data.

### Inaccuracies

Show what can go wrong if students don't measure correctly and in the same way. For example: Have students collect data about the diameter and circumference of lids, to investigate relationship. Ask: Why would it be difficult to plot the results in a scatter plot if everyone has measured in a different way? How does inaccurate measuring affect how clearly you can see relationships in the data? (Link to Summarise and Represent Data, Key Understanding 3.)

### Balloon Power

Ask students to consider whether their data represents the situation well enough or whether they have to think back to the situation to answer their question. For example: When considering which balloon-powered car is the most reliable, ask students to say whether a record of the average distance travelled by all the cars is sufficient to answer their question. Why? Why not? What data would you have collected if you knew then what you know now? (See Key Understandings 1, 2 and 3.)

### Consistent Data

Have students decide how they could ensure consistent data when recording more difficult or complex frequency data. For example:

- counting vehicles that pass the school could be shared so that one student counts cars, one counts trucks and the other bicycles and motorbikes
- when counting weeds on a section of the oval, the area could be marked into small squares and different students could count different squares
- to count the number of ants in a particular place, four students could all count at the same time, and then average their totals.

Ask: How can you ensure that you are recording all that you can see?

### Missing Answers

When students are organising data obtained from written questions, ask them to discuss what they should do about missing answers, or answers that are obviously incorrect. Ask them to consider the effects on their data and conclusions of including such information. For example: If they want to find out the mean pocket money that students in their class receive, and one child writes $2000 per week, would you include that response or not? What effect would this have on conclusions?

### Zero Values

Have students decide when it is or isn't appropriate to include values or measures of zero in data. For example: When collecting information about the number of children in a family, would you need to include families with no children? Why? Why not? Draw out that the decision to include zero values depends on the original question.

# SAMPLE LESSON 4

**Sample Learning Activity:** Middle —'More Scary Things', page 123

**Key Understanding 4:** We should make our data as accurate and consistent as possible.

## Teaching Purpose

My class of nine- to ten-year-olds had made many decisions in the organisation of their data in response to an initial question, *What scared you the most when you were younger?* However, the way their responses were recorded had been planned by me. I wanted to give them some experience in making decisions for themselves so they could see that the accuracy and consistency of their data could affect their conclusions.

## Action

The students decided that, although some of them were more scared of one thing than another, the list was fairly predictable—they had thought the same kinds of things were scary as did the Year 2 students who had also worked on the project. We wondered if adults would have been scared of the same kinds of things when they were children.

*I wanted my students to experience for themselves that 'asking people' is no simple matter and that problems may arise if data collection is not carefully planned.*

With a little prompting, the students decided they could collect some data to find out for themselves. Angelina suggested they could ask some adults that night and bring the information to class the next day. I hoped their data would provide me with the opportunity to draw out the need for more careful planning.

## Drawing Out the Mathematics

The following day I asked my students to work in small groups and talk about what they had found out. As I walked around the room I listened to what they were saying. I then asked students to say what they'd found.

Rhiannon: *I found out that my mum was scared of snakes just like I am, but Josiah's stepdad was scared of vampires.*

Elizabeth: *But vampires weren't on our list.*

Josiah: *Doesn't matter, that's what Dad said he was scared of.*

Elizabeth: *But you were supposed to ask if it was the same things. I gave Nana the list and she ticked what scared her.*

Rhiannon: *Yes, I thought we had to find out if they were scared of the same stuff—I just asked Mum if she was scared of snakes like me when I was little.*

Others joined in, arguing about what they thought they were supposed to find out. They realised that, although they assumed they all knew what had been needed, they had interpreted the big question differently. They decided they should have all asked, '*What scared you most when you were young?*' so they could 'match' their own responses to the same question.

Josiah: *But that's kind of what I asked my dad, 'When you were a kid what was the scariest thing?'*

Me: *And can you remember exactly what he said?*

Josiah: *He told me about when he was my age or a bit older, he saw a movie about vampires and he got really scared by it.*

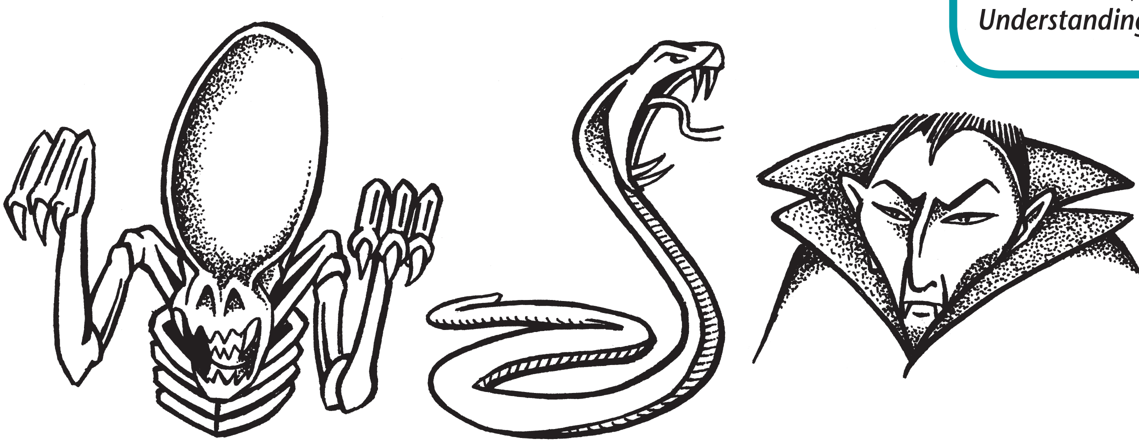
Peter: *But that means your dad was older than we are. I'm not scared of monsters now, only when I was in pre-school.*

This stimulated a lot of talk about just what was asked and how the adults responded. They also realised the data they'd gathered about adults had to stand alone. When they were categorising data about themselves, they had the opportunity to add and clarify what they had initially written. They couldn't do this with the new data. For example:

Aiden: *Uncle was scared of aliens, he said there was a movie called 'Aliens' and it was really scary, but he didn't say how old he was when he saw it.*

Michael: *Rhiannon's mum might have been MOST scared of something else, but Rhiannon only asked her if she had been scared of snakes when she was little, and we don't even know how old she was either.*

*Although in the initial class discussion we did not clarify what 'younger' meant, students often prefaced what they said with* **When I was in pre-school ...**, *or* **When I was in Year 1 ...**, *which tended to focus the rest on that age group when answering the question themselves. We can't make the assumption that 'we all know what we mean' when the data is collected from others. (See Key Understanding 1.)*



Through this conversation we were able to focus on how accurate their data was for answering our initial question, and the students themselves decided that there were too many problems for it to be useful. They wanted to go back and ask again. With help they formulated a question that they thought would elicit more accurate data: *When you were about five years old, what scared you the most?*

Once they were satisfied with the question, I drew attention to the idea of consistency. I asked: *Now that you have this question, do you think you will all remember it, and not change it at all when you ask it tonight?* This stimulated more discussion and they decided to type the question on the computer, print and photocopy it so the adults could write their answers on the sheet. That way, we'd be sure they would all answer exactly the same question.

## Reflection

The next day, when students brought back their responses, they discovered another problem that we had not anticipated. Some adults had written down several different 'scary' things. We didn't know which scared them the most, or if they were equally scared of several things.

This helped students to reflect on the need to carefully think through the way we produce our data, and, when asking people, to even anticipate possible responses, to make sure that the information we get back from our question is as accurate and reliable as it can possibly be.

### *Did You Know?*

Consider a watch that never loses or gains time but is always exactly five minutes fast. Anybody reading the watch would get an inaccurate (that is, invalid) measure of the time but a completely consistent (that is, reliable) one. The owner of the watch could rely on it always being five minutes fast. A watch that lost or gained time would be inconsistent or unreliable. Of course, such an inconsistent watch would also be invalid as a measurer of time because it could not be relied upon to be accurate either! This is why statisticians say that an instrument or measuring device can be reliable without being valid, but cannot be valid without being reliable.

Similarly, a test would be considered to be an accurate (or valid) measure of mathematics achievement if the data it generated truly reflected the mathematical learning we say we value. It would be a consistent (or reliable) measure of mathematics achievement if a student taking the same (or an equivalent) test in another setting would be expected to do equally well, and if different markers would come to the same conclusions.

# KEY UNDERSTANDING 5

## *Sometimes we collect data from a subset of a group to find out things about the whole group. There are benefits and risks in this.*

We can sample some of the people, some of the things, some of the time, some of the results … The essential idea in sampling is that we can make inferences about a whole group from data produced on a subset of that group. That is, we assume that the sample (subset) will behave in a roughly similar fashion to the population (whole group) and hence draw conclusions about the whole from the part. In doing so, we make statements about how confident we are in making those generalisations.

Considerable care needs to be taken in selecting samples from populations and in forming conclusions about populations from samples so that we *can* make clear statements about how confident we are of the conclusions drawn. However, the technical processes involved in selecting samples, determining sample size and making inferences from samples are not straightforward, and developing these more technical skills is not a goal for the primary years.

The essence of this Key Understanding for the primary years is twofold. Firstly, students should learn to distinguish between collecting data on a whole group (a census of the population) and collecting data on a subset of the group (a sample of the population), with the intention of drawing conclusions about the whole group. Secondly, they should consider in an informal way whether it makes sense to collect data from a subset of a whole group and if so, how they should choose the smaller group.

Distinguishing a sample from a population is not as obvious as it first seems. Sometimes, the population is infinite, as in the number of times you could toss a coin. Sometimes it is changing constantly, as in all of the people in Australia. And sometimes it is hard to determine the population a sample represents, e.g. What is the population to which we can generalise results from our class of Year 6 students? Students in the middle to later primary years should begin to distinguish samples from populations in reasonably straightforward situations. They should compare situations where a census is needed and those where a sample may be sufficient or necessary. For example, in investigating the insect population, they may develop strategies for sampling parts of the garden. In investigating music tastes, they may decide that surveying the Year 6 children in their school would give a 'pretty good' indication of music taste among 11-year-olds, but not older or younger children.

Students should also discuss things that might introduce bias into samples and consider ways of overcoming bias in the selection of samples. In doing so they could informally consider the three main ways that we construct samples. Firstly, we might construct stratified samples where parts of a sample are chosen from each of several groups such as gender, or grade level, or geographic location. Secondly, we might select random samples, as in drawing names out of a hat where every member of the population has an equal chance of being selected in the sample. And thirdly, we can select convenience samples in which we decide to work with the people, or objects, or times, readily available to us. The approach should be quite informal, with sampling activities ranging over these three approaches without students having to name them or distinguish them during the primary years. The question they should be asking informally is: How confident are we that the sample represents the population sufficiently well?

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Autumn Leaves

Have students collect fallen leaves during autumn and then group them to show which ones come from the same tree. Ask: Which trees drop the most leaves? Do some trees keep their leaves and not drop them? Does our grouping of the leaves show which trees we have more of in the school yard?

### Buttons

Ask students to group a collection of buttons according to how many holes in each. Ask: Does this mean that most people have two-holed buttons on their clothes? Draw out that looking at just a few buttons doesn't tell us much about all 'button wearers'. (See Key Understanding 2.)

### Eye Colour

Have students draw a picture of their eyes to show their eye colour and then group all the pictures to find out how many students have each colour. Ask them to consider the numbers in each group and to say whether the same number of students would have the same colour in different classes.

### *Did You Know?*

Telephone polls where people give a response by phoning a particular number can give completely misleading results. A telephone poll in the USA once asked people to phone in to respond to the question: *If you had the chance to live your life over again, would you have children?* The overwhelming response was 'no'. The government was so concerned that it commissioned a survey that asked the same question of a carefully stratified sample of the population. This time there was an overwhelming 'yes' response. One problem with self-selected samples, such as the telephone poll, is that there is no way to control for bias in the sample—and more than one call is often accepted from the same telephone. In this case the phone-in was invited at about 6.30 p.m.—a particularly difficult time of day for most parents. It's not difficult to see how that particular phone-in sample may have been biased towards a 'no' vote.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔

### Favourite Cereal

Present students with situations where we cannot collect information on everyone or everything of interest, and ask them to decide what to do. For example: Which breakfast cereal is sold the most? Ask: We can't find out what every single person in the country has bought this week, but what else could we do to find out? Who could we ask? Draw out that sometimes we have to collect a little bit of data and use it to give us an idea about what might happen in general.

### Hangman Extended

After playing Hangman (page 17) ask students how they could find out which letters are the most common. Ask: Can we check every single word ever written? What could we do instead? Where should the words come from? Should we look in a dictionary? Draw out that sometimes it is appropriate and sensible to collect a little bit of data, but we need to be careful about where it comes from.

### Sleepy on Sunday

After reading *Sleepy on Sunday* (Graham, 1989), ask students: Do you think that this is what most old people do during their week? How could we find out what most old people do on, for example, Sunday? If we asked all of our grandparents, could we say that we know what most old people do on a Sunday?

### Fair Sample

Invite students to decide if selecting a certain sample would be 'fair'. For example: We want to find out what sort of sports equipment to buy but only have time to ask 30 students. Should we just ask the boys? Just the Year 6s? What would be a fair sample?

### Pen Pals

When writing to pen pals about what typical Australian kids do, ask: Is what our class does enough of a sample? How would you find out what other Australian children do? Where would these children live?

### Comparing Data

Have students compare class 'where we were born' data with national figures. Ask: Is our class typical? Could we use our class as the representative for the country? Can you think of reasons for the difference? Draw out that often a sample doesn't necessarily represent the whole population.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔

### Sampling

Encourage students to research and brainstorm how 'samples' and 'sampling' are used in the world around us. Ask: When you hear the phrases 'taking a sample' or 'getting a sample', what do you think of? Draw out ideas of sampling through free samples, phone polls, tagging a sample of a certain species of animal to estimate their population, and samples taken during an election.

### Is It Fair?

Give students situations that would clearly be non-representative. For example: If I wanted to find out what sports the school should play on our mini-sports day, would asking the Year 6 boys what they like be a good way to get that information? Would that be fair? Why not? What part of the school population would that sample favour?

### Choosing a Sample

Have students explore different ways of selecting a sample. For example: We want to find out if Year 5, 6 and 7 students think they should or shouldn't line up before coming into the classroom. To avoid disrupting the classrooms we decided to ask 30 students during lunchtime to sample opinion from the six classes. We could just ask our Year 5 class, which would be convenient, or the first 30 people from any of those years we come across. We could put everyone's name in a hat and draw out 30 names, or we could choose a stratified sample of five (drawn randomly) from each classroom. Discuss which is preferable. Which is the easiest sample to choose? Which is the more representative sample? Which would give us the most confidence in our prediction? Have students carry out the survey in these different ways, and also survey all Year 5, 6 and 7 students. Compare the results with their ideas about which sampling strategy was best.

### School Representatives

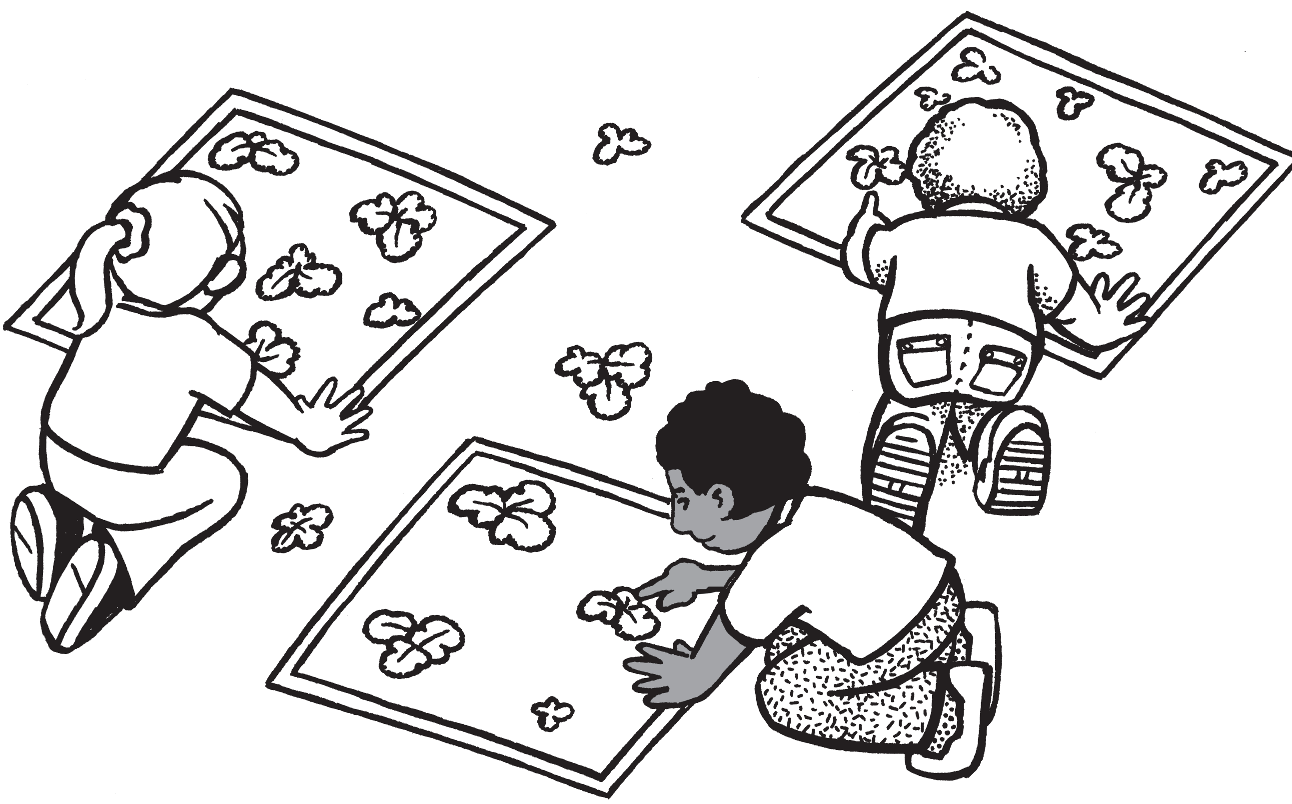
Ask students to discuss whether students in their class would be representative of the whole school for collecting data about a specific topic. Would it be 'fair' to just ask our class? Would a better sample be a few students from each class—the same number in the sample but more representative of the whole school? Why would you think this kind of sample is better? Would we also try to choose equal numbers of boys and girls in a sample? Why? Why not?

### Generalisations

After students collect data from their own class, ask: Is it reasonable to generalise? Could we say this is the case for all Year 5s? All students? Why not? What other data would need to be collected?

### Weed Growth

Discuss ways of sampling weed growth on the oval. Ask: Would it be sensible to count all the weeds on the whole oval? What could we do? Draw out that we could, for example, mark out a square metre, and count those weeds, multiplying the number by the total square metres on the oval. Ask: Would several squares in different parts of the oval be more representative of the overall weed growth?



### Making Assumptions

While carrying out chance activities, draw attention to the assumptions we make when we use data such as rainy days or accidents at school to make predictions about the future. Discuss how confident we are that our 'sample' (the data we have collected) represents the 'population' (all of what has happened in the past and future) to which it refers. Draw out that for this kind of 'population', we can only ever use a sample.

# SAMPLE LEARNING ACTIVITIES

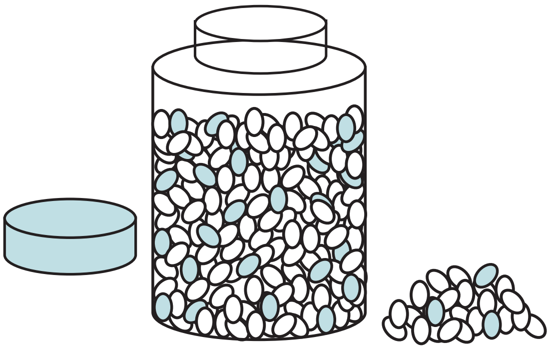
## Later ✔✔

### Counting Fish

Have students consider how it is possible for scientists to estimate the number of fish in a lake. Explain that they do this by catching, tagging and then releasing a set number of fish. Later they catch some fish again and work out what fraction of the catch is tagged. Because they already know the total number of tagged fish in the lake, they can use this relationship to estimate the total fish population. For example, they release 100 tagged fish. They later catch 100 fish and find ten of them are tagged. This is one-tenth of the caught fish, so they assume that the 100 tagged fish should be about one-tenth of the total fish population. This means the total number of fish in the lake should be about 10 times 100 (i.e. about 1000 fish).

Have students simulate part of this process using beans in a jar to represent fish in a lake and consider how useful such a strategy might be. As a class, help students count out 800 white beans (total fish in the lake), then colour 200 of those (a quarter of the beans) with a permanent felt pen (tagged fish). Put all the beans in a large jar and mix them well. Have students take turns to 'catch' a small number of 'fish' (i.e. grab a small handful of beans without looking). They count how many 'fish' they have caught and check how many of them are 'tagged fish'. We know that the true fraction of tagged fish is one-quarter (200 out of 800), so we can check how close our different samples are to this. Ask: Are about a quarter of your fish tagged? What if you put your catch together with your partner, or your group?Are the tagged fish about a quarter of the total catch now? How many fish do you think we would need to catch to give an accurate estimate of our fish population?

Repeat the process using different sized samples to test their suggestions. Draw out that for small samples, the results vary a lot, but for larger samples the fraction of tagged fish will be closer to the true relationship. Note: This strategy only works if the tagged fish are well mixed in with the rest of the fish. (Link to Understand Chance, Key Understanding 7.)



CHAPTER 5

# Collect and Process Data (Part B)
## Summarise and Represent Data

This chapter will support teachers in developing teaching and learning programs that relate to Part B of the outcome:

***Summarise and represent data for effective interpretation and communication.***

### *Overall Description*

Students summarise and represent data produced by themselves and others. They describe patterns in data and make concise but meaningful summaries using statistics to describe proportions, averages and variability. They understand that none of these statistical tools are ends in themselves—they are useful only in so far as they assist interpretation and communication. They choose and use diagrams, tables, plots and graphs that are suited to the kind of data and the purpose of the display.

They understand, on the one hand, that quickly produced displays can be very informative both for their own understanding of the data, and when trying to give others a 'snapshot' impression of trends and relationships. On the other hand, there are times when precision is essential so that they and others do not misinterpret, arrive at erroneous conclusions or mislead. Therefore, they realise that data can be distorted accidentally or deliberately to reach inappropriate conclusions. They consider the impact of technological change on the handling of data and consider ethical issues in the representation of data, and act responsibly in this regard.

| Markers of Progress | Pointers<br>Progress will be evident when students: |
|---|---|
| Students display objects and pictures, and describe data in words and numbers. | • display classified objects in order to compare collections, e.g. string red beads and green beads separately to decide which collection has more<br>• display in one-to-one correspondence, pictures or objects that represent themselves, e.g. place in rows (one each for car, walk, bus) pictures that show how they came to school<br>• draw a picture as a record of their results, e.g. draw a picture of the 'graph' they made with actual pieces of fruit<br>• summarise information by counting, e.g. count how many children have caught a fish and say *Eleven children have caught a fish*<br>• talk about what they have found from their data collection and display, e.g. *We lined up the fruit in rows. There were more apples than bananas* |
| Students display and summarise data based on one-to-one correspondences between data and representation. | • suggest ways of efficiently counting how many there are in each of several categories, e.g. use coloured buttons to represent students' snack preferences and arrange buttons in one-to-one correspondence<br>• compare heights (or lengths) of the columns in a block graph to place categories in order, e.g. *This shows more students have a digital watch than one with hands,* and *The fewest have no watch*<br>• make block graphs using 'real' data, e.g. students line up by the month in which they were born; line up beans used to cover various shapes<br>• make graphs and plots using one-to one correspondence between 'real' data and a representation, e.g. glue on a red square for each time red appears on the spinner<br>• understand the need for a baseline and space blocks regularly (in provided grids) to allow comparisons to be made<br>• place direct measurement data in sensible sequences using a baseline, e.g. cut paper strips to fit around their heads, write names on the tapes, and make a bar (column) graph by lining up the bottom strip |
| Students display and summarise data using frequencies, measurements and many-to-one correspondences between data and representation. | • use diagrams such as Venn diagrams and two-way tables to represent a two-way classification<br>• summarise data based on tallying, e.g. use a conventional tally method to record the number of times a thumb tack falls on its side or on its top, and summarise by counting the tally for each group<br>• summarise data in diagrams and tables that show frequencies for different categories, e.g. the category may be type of food, and frequency the number of students who chose that type; names of children recorded in a Venn diagram are replaced by the count of how many were in that category<br>• use lengths to represent other measures such as time or mass, e.g. mark paper strips with the 24 hours of the day, shade the time between going to bed and getting up, cut shaded parts and make a graph of the times in bed<br>• display data in pictographs where each symbol represents more than one unit, such as one picture for each ten children<br>• display frequency data in (vertical and horizontal) bar graphs where one axis shows the whole numbers (0,1, 2, 3, …) |
| Students display frequency and measurement data using simple scales on axes and some grouping, and summarise data with simple fractions; highest, lowest and middle scores; and means. | • represent data in diagrams and tables that may include arrow diagrams, Venn diagrams and two-way tables<br>• make quickly produced 'working' graphs to explore data, e.g. use sticky paper notes with food choices written on them to make quick block graphs based on different ways of classifying foods<br>• realise that it is sometimes helpful to group data involving whole numbers into class intervals, e.g. having estimated the number of sweets in a jar, organise the estimates in intervals such as 41-45, 46-50, 51-55, 56-60, to compare estimates with the true amount<br>• use fractions and decimals to summarise data, e.g. about 0.4 of Year 2 students ride to school, 0.3 walk, the rest come by car; the thumb tack fell on its side about two-thirds of the time<br>• find the mean where there is sufficient data to make summarising sensible, e.g. the average height of the girls in class is 149 cm<br>• put data in order and describe the highest, lowest and middle scores<br>• use the mean to get an estimate of a number, e.g. use the mean number of raisins in 25 boxes to estimate the mean |
| Students display one-variable and two-variable data in tables and plots and summarise data with fractions, percentages, means and medians. | • display measurements in tables with provided class intervals, e.g. find the mass of various rocks and group the data from 0 to 250 g, more than 250 g and up to 500 g<br>• display one-variable data in dot frequencies with various scales, including multiple and decimal fractions, e.g. make a dot frequency of the actual time span for each person's estimate of 30 seconds<br>• use stem plots (or stem and leaf graphs) to group and display one-variable data, e.g. make a stem plot of the ages of their parents or caregivers<br>• represent two-variable data in scatter plots and make informal statements about relationships, e.g. between time spent reading and watching TV each week<br>• use means or medians to summarise data where there is sufficient data to make summarising sensible, e.g. of the number of class hours spent watching TV<br>• use fractions or percentages to compare data, e.g. *Before, I got 26 balls from 50 tries, that's 52%. This time I got 24 from 40 tries, or 60%. They are close, but I may be improving a bit* |

## Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

| Key Understanding | Stage of Primary Schooling—Major Emphasis | KU Description | Sample Learning Activities |
|---|---|---|---|
| **KU 1** We can display data visually; some graphs and plots show how many or how much is in each category or group. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 140 | Beginning, page 142<br>Middle, page 145<br>Later, page 149 |
| **KU 2** We can display data visually; some graphs and plots show how one quantity varies over time. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 158 | Beginning, page 160<br>Middle, page 162<br>Later, page 164 |
| **KU 3** We can display data visually; some graphs and plots show how two quantities are related. | Beginning ✔<br>Middle ✔<br>Later ✔✔ | page 166 | Beginning, page 168<br>Middle, page 169<br>Later, page 170 |
| **KU 4** We use tables and diagrams to organise and summarise data in a systematic way. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 174 | Beginning, page 176<br>Middle, page 178<br>Later, page 180 |
| **KU 5** How we display our data depends on the kind of data we have and our purpose. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔ | page 182 | Beginning, page 184<br>Middle, page 185<br>Later, page 188 |
| **KU 6** We can use words and numbers to summarise features of a set of data. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 194 | Beginning, page 196<br>Middle, page 197<br>Later, page 199 |

**Key**

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

## KEY UNDERSTANDING 1

***We can display data visually; some graphs and plots show how many or how much is in each category or group.***

The first aspect of this Key Understanding is that we can display data visually. It is common to Key Understandings 1, 2 and 3 and develops gradually from related work for all three. The second aspect of this Key Understanding relates to the plots and graphs used to represent one-variable data where a frequency or measurement is the thing that varies, as in line plots, stem plots, pictographs, bar graphs, histograms and pie graphs. With the exception of pie graphs, these are based on columns or rows, each labelled with a category or group, where the length of a column or row represents the frequency or measurement associated with that category or group.

Initially, the need to use a common baseline and space objects evenly to ensure a match between number and length will not be obvious to students. To understand this they will need plenty of opportunities to make their own displays using actual objects or pictures that they have made or collected, rather than simply filling in spaces on provided grids. Through exploring what their displays show (or seem to show), they will gradually see the need to think about appropriate placement.

Using one thing to 'stand in' for another is not obvious and students need help to make the transition from displaying actual things to representing these with tokens or pictures. For example, after lining up according to eye colour, they might each write their name and draw their eyes on a sticky label, and use one-to-one correspondence to build up a graph. Such pictographs are a way for students to begin to abstract or simplify information and this development should not be rushed. Students often continue to want to show the identity of each piece of data in their displays but, with help, they will gradually learn to represent data where information about individual values is increasingly summarised and therefore some information becomes lost or obscured. For example, they can replace their personal drawings with a more abstract cross above the appropriate column on a line plot, or colour in a corresponding square to form a block graph. As they progress through the early and middle years, students should represent data with increasingly higher levels of abstraction. For example, they should make block, picto- or bar graphs using one square or one symbol to represent more than one unit (e.g. one square for ten people), and make bar graphs to represent other measures such as time or mass.

There is a conceptual leap required for students to begin using the vertical scale to produce bar graphs directly from frequency data, rather than colouring or counting squares one at a time. They must switch from seeing the data as individual pieces of information to seeing it as aggregate information about a group and understand that length is used to represent an amount.

In the later primary years, students should also learn how, and for which purposes, stem plots and pie graphs are constructed. They should use bar graphs for grouped data and with more complex scales on axes. They should also make use of computer graphing programs to investigate the effect of varying the groupings or the type of graph on the impression we gain of the data.

## Progressing Through Key Understanding 1

Initially students can display their collections physically or using their own pictures. As students continue to progress they can represent one thing with another and display their data in different ways. Next they can represent more than one thing with squares or pictures and make use of a simple vertical scale to produce bar graphs from frequency data. They know that lengths in bar graphs can represent other things besides length, such as mass or amount of money raised. As students progress further they can appropriately represent categories in their bar graphs as either discrete or continuous, and use frequency scales based on multiples. Later, students can use quite complex scales on axes to produce the full range of graphs required.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔

### Student Groups

Have students group themselves into various categories such as wearing or not wearing red; likes or doesn't like watermelon. Students then line up in their groups and make comparisons using matching and one-to-one techniques. Ask: How can we tell if one group has more than another? Can we tell just by looking, or do we need to count?

### Birthdays (1)

Form groups of students with the same birth month. Using a rope tape on the floor or a wall as a baseline, have students line up in their groups. Take a digital photo of this display, then ask students to look at the photo and say which month has the most students' birthdays. Ask: How can we find out? Can we tell by just looking, or do we need to count?

### Birthdays (2)

Draw up a large sheet of paper showing the months of the year. Ask each student to write their name on a card, then place their card on the paper, on their birthday month. (Allow them to place the cards in a bunch if they choose.) Ask: Which month has more birthdays? Are there months with no birthdays? How could we place the cards on the sheet so that we could tell just by looking? Help them reorganise the cards into evenly spaced rows and columns. Ask: Do we need to know which card belongs to which person?

### Birthdays (3)

After the previous activity, have students use sticky notes with their names on to produce a class block graph representing the month in which students' birthdays occur. Have each student take back their sticky note, replacing it with a cross as the card is taken off, producing a line plot. Ask: Can we use the crosses to tell us which month has the most/least birthdays? (See Collect and Organise Data, Key Understanding 2.)

### Physical Graphs

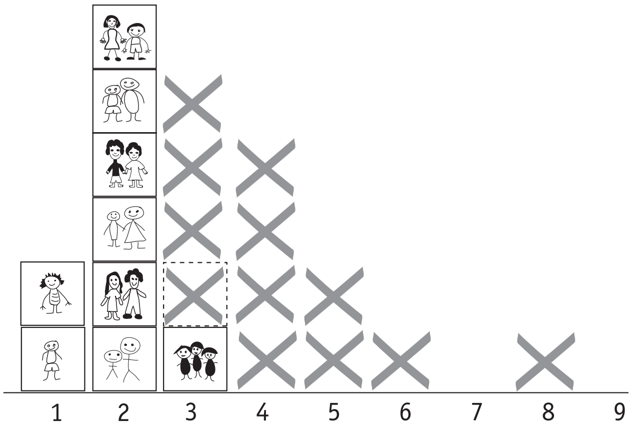
Use Unifix cubes to create physical class graphs for data of interest, such as favourite colours. Initially, allow students to choose cubes of the colour they like best and construct a graph to show the most-liked colours in the class. Then collect other data, such as the type of fruit brought in that day, and ask students to assign a colour to a category, and build up a physical graph.

## Colour Graph

Have students consider how to position objects of different sizes when lining them up to show how many in each category or group. For example: Repeat the previous activity, replacing Unifix cubes with different sized coloured blocks (small green blocks, large yellow blocks, etc.). Help students line up the blocks to find out which colour is the most popular. Focus on the need to space the blocks evenly. (See Sample Lesson 1, page 154.)

## My Family

Invite students to use a line plot as a quick way to represent data. For example: Have each student draw the children in their family on a sticky note, then place the note on a line plot showing numbers along the base from 0 to beyond the largest number of children. Ask each child to remove their note and put a cross in its place, to produce a line plot. It may help to have a discussion of what a family is first, since this will be culturally specific.




## Comparing Heights

Cut lengths of paper tape that match students' heights. Arrange a number of these tapes on the board, placing them vertically so that they still represent height, but using different starting points that make it difficult for students to compare the lengths. Ask: Can you tell from our display who is the tallest? The highest tape belongs to Sally, but she isn't as tall as Fiona. How could we make it easier to see quickly who the tallest person is? Rearrange the tapes so that they begin on the floor (and therefore are as high as the students). Ask: Is it easier to see now? Then rearrange again so that they are lined up with the bottom of the board. Ask: Can we still tell who is tallest?

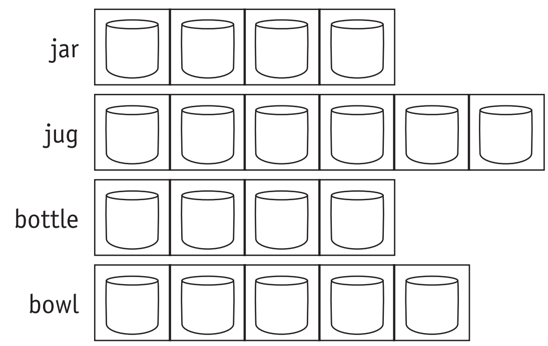
## Tree Size

Repeat the previous activity, using a horizontal layout instead of vertical. For example: Use strings to measure the distance around trees in the playground then ask students to pin the strings on the display board so they can compare the lengths. Allow them to choose how they pin them up. If they pin them at angles and with different starting points, ask: Can we see which is the longest/shortest when they start in different places or are crooked? Why? Draw out the need for all to start on the same line and all to be evenly laid out.

## Beginning ✔✔

### Measurements

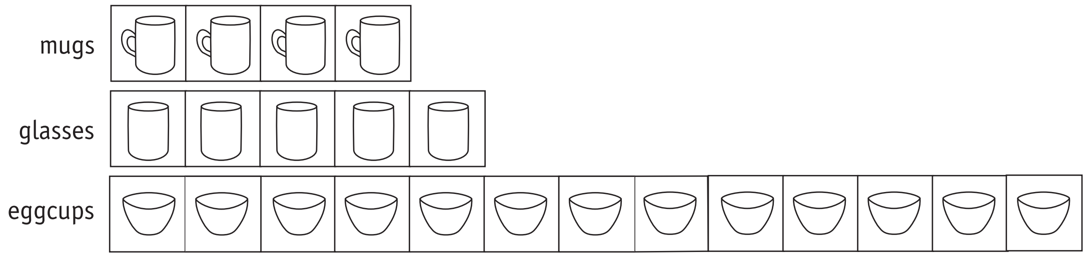
After students have recorded their measurements of different objects using the same unit, they produce a picture or block graph of the data. For example: Have students use a cup as a unit to measure the capacity of a variety of containers. They then draw a picture of a cup on a number of squares of paper, according to how many cups each container holds. They arrange the squares of paper next to a label for each container tested. Ask: Can you use the graph to say which container held the most, without counting each cup? (See Interpret Data, Key Understanding 2.)




### Comparing Units

Ask students to measure one object using different units, and produce a picture or block graph of their data. For example: They use various objects to fill a bowl with water. Ask: Which object did we need to use the most times to fill the bowl? (See Key Understanding 3, and Interpret Data, Key Understanding 2.)

**The bowl holds:**



# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Block Graphs

After students have made physical displays of data (see 'Beginning' activities, pages 142 to 144), have them represent what they have done on paper by drawing pictures or listing names. Compare their representations with the original physical display. Ask: Can you tell just by looking at your drawings which fruit we had the most of? On Sally's graph it looks like we had more bananas, but on Jane's, it looks like we had more oranges. If we look at our fruit, we had more oranges. How could Sally change her graph? Draw out the importance of spacing the pictures evenly. Ask students to do another graph of their data, but this time provide squared paper and ask them to draw pictures, or write names, one to each square.

### Block Graphs Extended

Extend activities like the ones above where students draw a picture or write a name for each object, to have them use more abstract representations of data. For example: Use Unifix cubes to create physical class graphs for data of interest. Have students assign a category to a colour, e.g let the blue blocks be the plums, and together build up a physical block graph. Encourage them to show on paper what they see in the 3-D graph. Ask: What needs to stay the same on the paper? What could be different?

### How Many Dogs?

Extend activities like the ones above by focusing attention on the number of students in each category before they draw their graph. Use large grid paper and ask: How many students have pet dogs? How many blocks will I have to shade above 'dogs' to show this?

### Block Graphs from Data

After students have shaded squares to match a physical graph, ask them to produce graphs directly from unsummarised data. For example: Collect the names of students who fit into specific categories (have no pets at home, one pet, two pets, etc.). Provide students with squared paper and help them to produce a horizontal line under which they write the names of the categories. They then shade squares one-to-one to represent each piece of data in a block graph, proceeding through a series of levels of abstraction over time and with various topics:

- Initially they name each square above the appropriate category for the relevant student and produce a block graph by shading named squares.

## Middle ✔✔✔

- Next they cross off the students' names one at a time as they shade or mark a square. The result will be a block graph in which individual students are not identifiable.
- Next they count how many students are in each category, and shade the appropriate number of squares.

In each case, students name their graph and the horizontal axis.

### Block Graphs with Scales

Extend the previous activities by providing students with squared paper on which a vertical and horizontal axis have been drawn and a simple (e.g. 0, 1, 2, ... 10) scale has been marked on the vertical axis. Repeat the sequence of levels in the activity above, but in each case link the number of squares shaded to the number on the frequency scale. Children name the graph and label each axis. The frequency axis says 'number of (children)'.

### Make Your Own Scale

Extend the previous block graph activities. Provide students with squared paper, and ask them to produce a block graph by one-to-one shading. Ask them to draw in the vertical axis so that another person could see how many were in each group without having to count.

### Using the Scale

Provide students with pre-formatted scales as in 'Block Graphs with Scales'. Ask them to count how many are in each category from their original data and use the scale to work out how high to shade without actually counting the squares one by one. Start with a context in which there are nine or ten squares in some categories (it is tedious to draw one square at a time, and mistakes are likely to occur). Draw out the efficiency of using the scale to determine the number of squares.

### Bar Graphs (1)

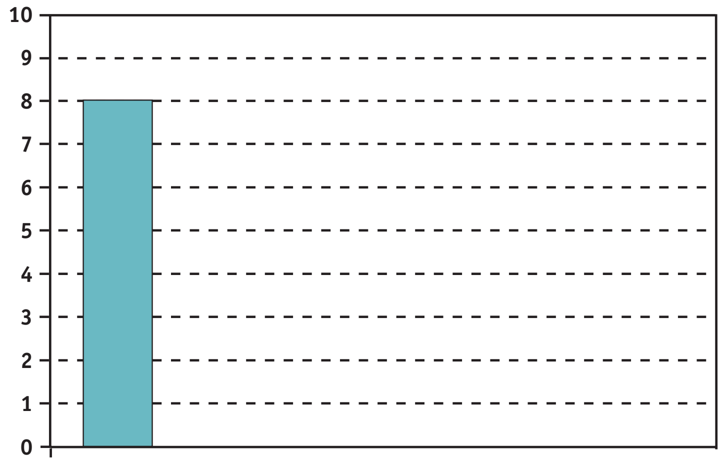
Have students construct simple bar graphs on grid paper from a tally, using the scale on the vertical axis to define the length of each bar, rather than counting squares. Help students to focus on the frequency scale and the length of the bar used to represent the data, not the individual squares. Ask: How can we see the number of squares without counting them?

### Bar Graphs (2)

Following the previous activity, change the axis to make 1 square = 2, so students need to take account of this length. Ask: What would happen if we just counted the squares on this graph? How can we work out where 5 finishes when each square means 2 things? What if we made 1 square equal to 5 things? What would change in the graph? *(The length of the vertical axis)* What would still be the same? *(How many things the bar represents)*

## Bar Graphs (3)

Provide students with a pre-formatted axis on paper, where the squares are not visible but lightly dotted horizontal lines are.




Ask students to shade columns to the appropriate heights using the vertical scale. Initially, all counting numbers should be on the scale, so students do not need to read between calibrations. Make the point that we usually make the bars the same width to show that the height relates to the different amounts.

## Bar Graphs (4)

Have students construct bar graphs with scales calibrated every 2 or 5. Ask them to work out how high to shade a frequency of, say, 3. Repeat with scales calibrated every 2 or 5, but no squares showing, that is, only horizontal dotted lines at labelled calibrations.

## Measuring My Length

Ask students to produce bar graphs displaying actual lengths. For example: They cut paper tapes to fit specific body parts (head diameter, length of little finger), write their name on the strip and produce a bar graph to display head diameters (or finger length) of students in their group of eight, and then in the class as a whole.

## Graphing Measurements

Have students collect measurement data other than length and construct bar graphs in which the height of the columns represents the measurements. For example: Ask students to fill identical paper cups with different materials (flour, water, sand, metal shavings) and find the mass in grams. Help them to choose a scale, perhaps 1 cm to 50 g, and cut paper strips to represent the weight of each material. Use the strips to produce a bar graph.

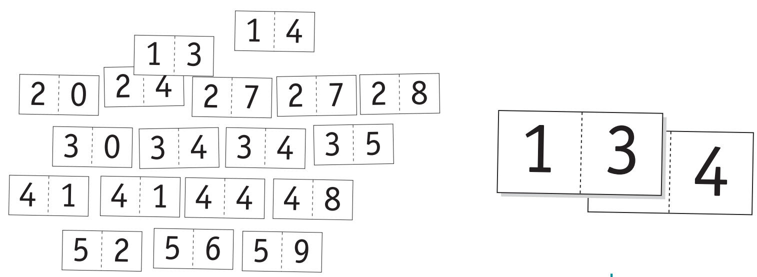
## Measurement Bar Graphs

Extend activities such as the two previous ones, by helping students to make a vertical scale to represent the measurements (e.g. label each centimetre as 50 g). Ask students to make a bar graph of their data by reading across from the vertical axis and drawing columns of a suitable height.

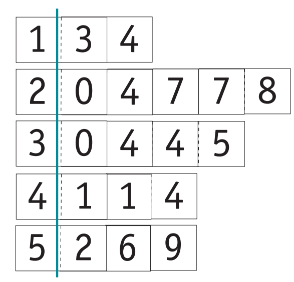
# Middle ✔✔✔

## Stem and Leaf Plots

Have students produce some data that results in a wide range of two-digit numbers, e.g. each student throws a ten-sided die ten times and adds the results to find their score; time how many seconds each student can balance a pencil on their fingers. Have each student write their score on a rectangle of paper with the tens digits on the left and the ones digit on the right. As a class, sort the rectangles into the tens and put the groups in order.



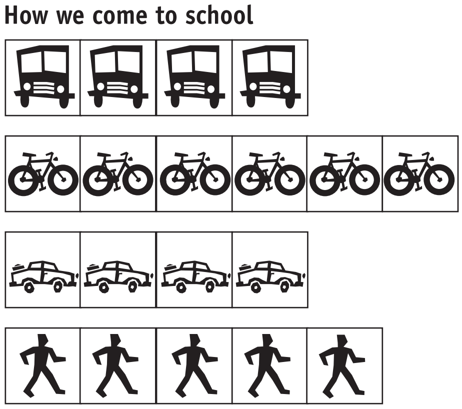

To construct a stem and leaf plot, paste the rectangles into strips for each group of tens, gluing the 'ones' part of the first rectangle on top of the 'tens' part of the next rectangle. Glue each strip onto a page, lining them up in order of the tens. Draw a line down the page between the tens and ones to complete the stem and leaf plot. Explain that the 'stem' is the tens digits, and the 'leaves' are the rows of ones digits. Ask: Can you still say what all the different scores were, even though parts are covered? What does the four in the first row mean? What does the four on the left of the fourth row mean? Can you see from the plot where most of the scores are? Do you think you could create a stem and leaf plot on grid paper from the scores alone, without any cutting or gluing?




## Pictographs

Find examples of pictographs used in newspaper and magazine reports and use them as a guide for students to construct their own pictographs for suitable data. Ask: What sort of little pictures get chosen? How many things are represented by each little picture? Have students draw, duplicate and cut out pictures to represent data produced from class surveys and produce their own pictographs.

**How we come to school**



# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Frequency Data

Have students produce bar graphs of frequency data based on their own investigations, without props such as pre-formatted axes. Model the appropriate choice of graph names and labels on each axis. Draw out that with frequency bar graphs, one axis shows categories while the other shows the frequency (number) in each category, e.g. This axis should be labelled 'number of …'. Show students that the bars can be vertically or horizontally laid out. Ask: How does this affect how the axes are labelled?

### Large Frequencies

Ask students to produce bar graphs on grid paper from frequency data, such as census data on the Internet, where the frequencies are sufficiently large to require a calibrated scale that goes up in numbers other than 1, e.g. 5 or 10. Ask: What is the largest frequency? What would be a good scale to make the graph fit the height and width you have chosen for it?

### Measurement and Price

Invite students to produce bar graphs of measurement or price data, based on their own investigations, e.g. the results of a survey to find the average height of the students in each grade level at school, or the cost per gram or per kilogram of various foods. Draw out that one axis shows categories while the other shows the measurement or cost. Say: This axis should be labelled 'weight in kilograms' (or 'price in dollars') to indicate the quantity. Have students choose a suitable calibration, thinking about the size of the graph wanted, and the largest quantity they need to represent.

### Pictographs

Have students construct their own pictographs for suitable data. Ask: What should we do when we want to show a part of a quantity? How can we decide how much of the little picture to show?

### Graphing Predictions

Invite students to use a graph to organise the predictions they make. For example: What do most of us predict will be the number of babies in the guinea pig's litter? On a class line plot constructed on grid paper, students fill in a square above the number they predicted and also write their number next to their name on a class list. Use the graph to see what the majority of the students predicted. Ask: Can we just look at the class list and get the same information at a glance? Why not?

## Later ✔✔✔

### Vertical Meaning

Graph data to help students understand that the height of the vertical bars need not reflect the height of things in the real world. While 'up' always means more, in the real world this might be 'lower', as in more depth or more depressed. Have students graph a range of measures such as time, depth, mass and strength of feeling, as well as create their own scales for other attributes. For example, in science they could develop a scale to rate the absorbency of different types of paper, or the strength of different kinds of thread. Have them create a bar graph using the vertical axis for their absorbency or strength scale, and listing the types of paper or thread along the horizontal axis. Ask: What does the length of the bars mean?

### Finding the Key

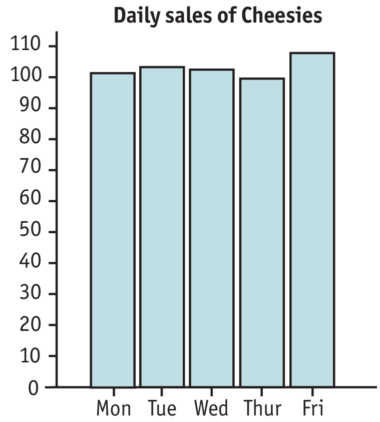
Have students inspect bar graphs and pictographs for the 'key' to interpret the colours, shadings and pictures. Ask: Have you included a key on your graphs? Encourage students to experiment with ways to do this and then assess ease of reading.

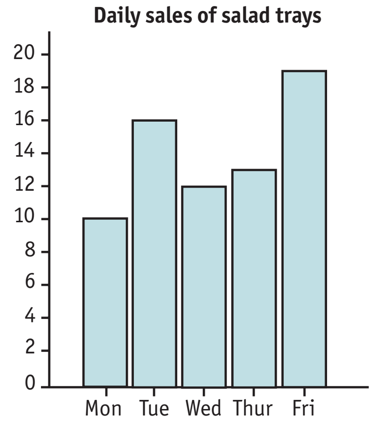
### Scale and Labels

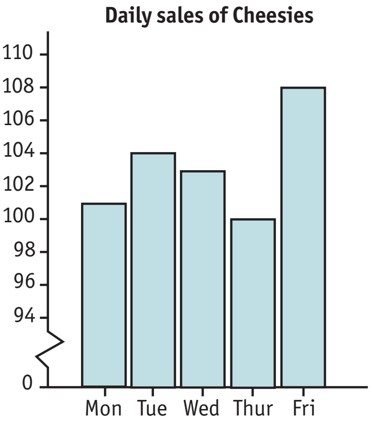
Ask students to inspect the scale and labels on the frequency axis of published bar graphs and use this information to help create graphs from their own data sets. Ask: What is the highest value in your data? How can you make sure all your data will fit on your graph? How will you segment the axis?

### Comparing Graphs (1)

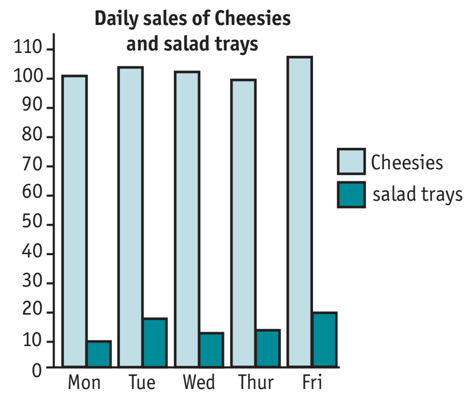
Have students construct graphs to compare two sets of data that are very different in quantity but not in range, e.g. food sales over a week at a school canteen. Have one set of data showing sales between 10 and 20, the other showing sales between 100 and 110. Ask students to construct a bar graph for each, adjusting the vertical frequency scales to be the same length. (See graphs, page 151, left and middle.) Ask: Can you see from each graph the quantity sold each day? *(Yes)* Do the graphs enable you to easily compare the daily variation in sales for the two foods? *(No)* Do they give the impression that salad tray sales vary more than Cheesies sales? *(Yes)* Could we draw a graph that better illustrates the variation in sales for the two foods? Have students redraw the Cheesies graph, using a scale from 90 to 110, and introduce the convention for showing an incomplete scale. (See graph, page 151, right.) Ask: How do the graphs compare now? What is emphasised? *(The similarity in the pattern of variation)* What is obscured or less obvious? *(The difference between the quantities of each food sold)* Draw out that this graph could be misleading because at first glance, the total sales for the two foods also look similar.




Have students then construct a graph with side-by-side columns showing the sales of both foods each day. (See 'Comparing Data', page 152.) Ask: How has this display made both the total sales and the range of variation easier to interpret?




## Comparing Graphs (2)

Extend the previous activity by asking students to find bar graphs in newspapers, magazines and on the Internet and having them redraw the graphs, changing the scale in different ways to see the effect on the overall impression gained by the graph. What is emphasised if we don't begin the scale from zero? *(The differences between the categories)* How might this distort the data? How might this clarify the data?

## Grouped Data

Invite students to produce bar graphs for data that is grouped, but which they can think of in categories, e.g. measure students' lengths from waist to knee, then group the measures into short, medium and long for bloomers to be made for the school production. Draw out that each category contains a range of leg lengths but the bloomers will be cut to one length for each size range. We can represent them in our graphs as three categories. (Here, the vertical bars are kept separate.)

## Histograms

Extend the previous activity to introduce simple histograms (see Background Notes, page 208), where the horizontal axis is a continuous scale for the range and the vertical bars touch to indicate the range of measurements included in each group. Thus the previous data could be grouped as 20–29 cm, 30–39 cm, 40–49 cm and represented in columns accordingly. Note that some groups of data in the range may have zero frequencies.

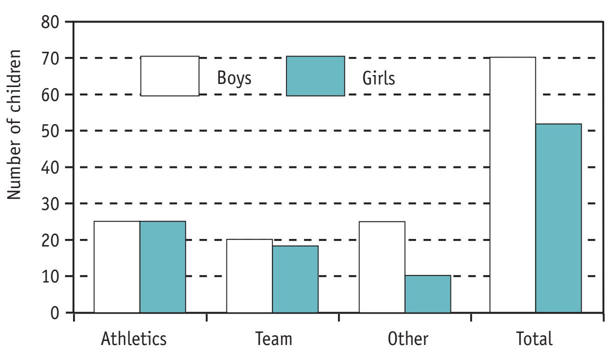
# Later ✔✔✔

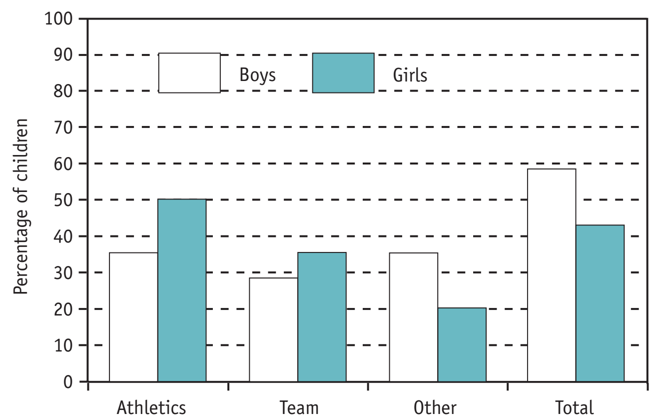
## Bars and Histograms

Obtain a range of bar graphs and histograms from various sources, including students' own constructions. Ask students to sort the graphs into those in which the columns or bars are separate and those in which they are touching. Ask: Can you write instructions as to when it is appropriate for the data to be represented by bars that touch? Do you think any of the graphs might be incorrect? Why?

## Comparing Data

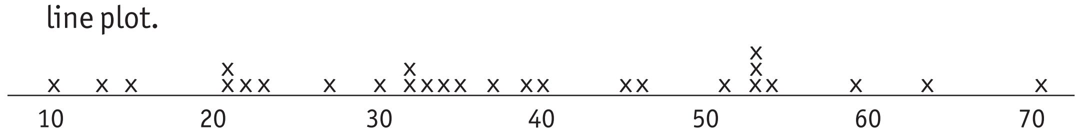
Ask students to use a computer graphing program to create graphs that compare two different sets of data. For example: After surveying all the Year 7 boys' and girls' preferences for sporting activities, students make a bar graph that compares boys and girls. Show how the bar for girls can be distinguished from the bar for boys, using a key, and consider if percentages rather than frequencies are appropriate. Draw out that percentages are needed when the total number of boys surveyed is different from the girls.

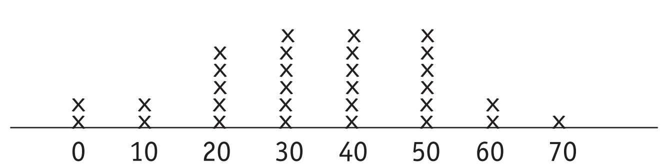



## Stem Plots (1)

Have students represent suitable data in a stem and leaf plot, and compare it to the same data shown in line plots. Ask students to construct a line plot for data that has a range of at least 50, for example, faction or house points earned by their class. They then round the data to the nearest ten and create a second line plot.



Help students to then produce a stem and leaf plot, first listing the 'tens' digits down the page and then adding the 'ones' digits of each student's points to the correct row. Students then sort the digits within each tens row to create the stem and leaf plot. Draw out that the list of tens digits forms the 'stem' and the 'leaves' are formed by the rows of ones digits. (The stem can continue beyond nine 10s to ten 10s and so on, for data into the hundreds). Each piece of data can be read by combining the tens digit in the stem with one of the 'leaves' (a ones digit). Ask: *What can you tell about the data in the stem and leaf plot compared to the line plot? What can you no longer see in the plot of the rounded points? Why does 'rounding to the nearest ten' give a different graph profile than is produced in the stem and leaf plot?*

| | |
|---|---|
| 0 | 4 3 |
| 1 | 5 0 3 |
| 2 | 1 1 7 4 3 |
| 3 | 9 3 7 0 2 2 4 5 |
| 4 | 0 6 5 |
| 5 | 3 4 1 9 3 3 |
| 6 | 3 |
| 7 | 0 |

| | |
|---|---|
| 0 | 3 4 |
| 1 | 0 3 5 |
| 2 | 1 1 3 4 7 |
| 3 | 0 2 2 3 4 5 7 9 |
| 4 | 0 5 6 |
| 5 | 1 3 3 3 4 9 |
| 6 | 3 |
| 7 | 0 |

## Stem Plots (2)

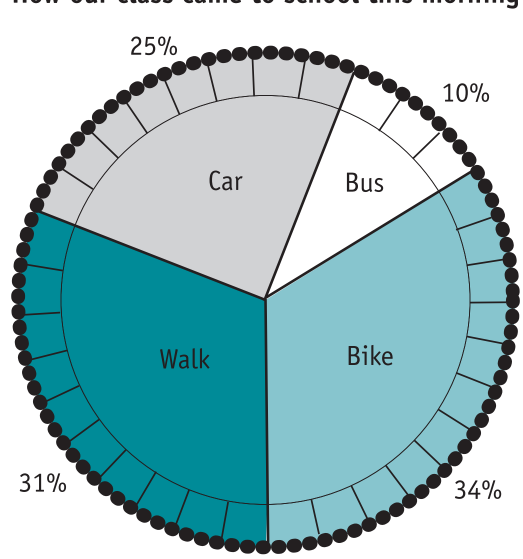
Have students compare two sets of data in a back-to-back stem and leaf plot. Note that the 'tens' digits form the 'stem' down the centre for both sets of data. The 'ones' digits are listed to the left for one set of data, and to the right for the second set of data. For example, the stem and leaf plot shown here compares Year 6 boys and girls for the number of points they have earned for their factions or houses. To find each piece of data, combine the tens (stem) with the ones (leaves). Ask: *Can you read the number of points each girl earned in the twenties row? (20, 24, 24, 27) How many did each boy earn in the twenties row? (21, 21, 23, 24, 27) How is this way of displaying data more helpful than using a table?*

| Girls | | Boys |
|---|---|---|
| 2 1 | 0 | 3 4 |
| 6 | 1 | 0 3 5 |
| 7 4 4 0 | 2 | 1 1 3 4 7 |
| 8 8 4 2 2 1 1 | 3 | 0 2 2 3 4 5 7 9 |
| 3 0 | 4 | 0 5 6 |
| 9 4 4 4 3 1 | 5 | 1 3 3 3 4 9 |
| 7 6 1 1 1 | 6 | 3 |
| | 7 | 0 |

## Pie Graph

Help students use whole-class data, such as ways of travelling to school, to construct a pie graph that shows how percentages relate to the 'slices'. First, string together exactly 100 beads and tie to form a circle. Cut a strip of card the same length as the bead circle's circumference and divide it into the same number of segments as students in the class. Colour the segments to match the number of students in each category. Draw a circle with the same circumference and place the cardboard strip around the edge to mark off the categories. Join the marks to the centre and then use the bead circle to approximate the percentage of students in each category.

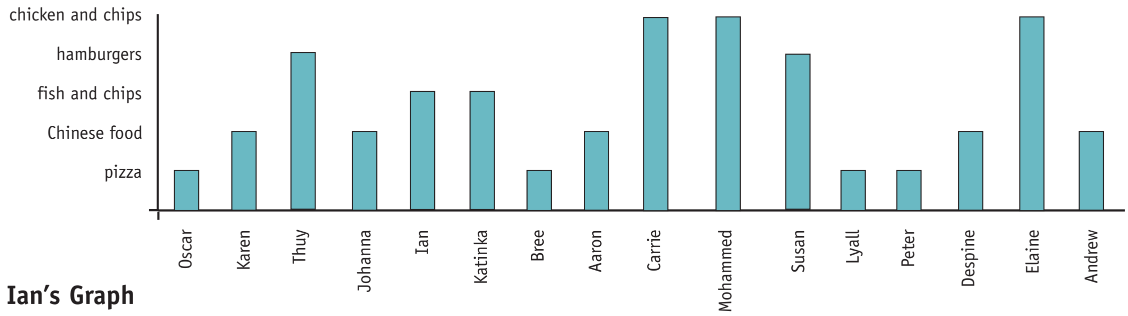
**How our class came to school this morning**



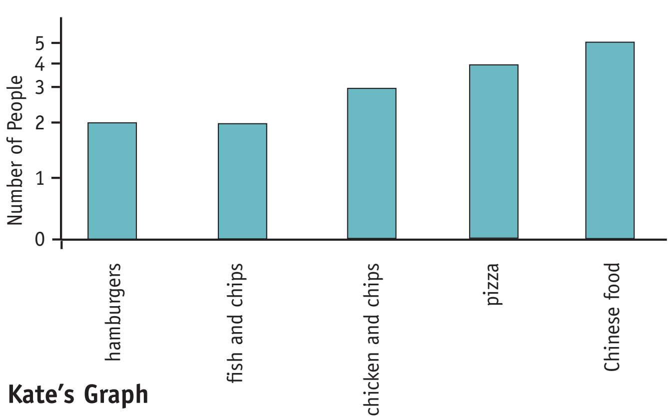
## *Did You Know?*

Ian and Kate (Year 7) produced these bar graphs, using the same data on their group's preferences for take-away food.

In Ian's graph we can see Aaron's favourite take-away food and find the frequencies by counting the bars that are the same size. So, if Ian believes the bars in his graph are meant to simply 'point' to or label the type of take-away food liked by each student, it is not surprising that he thinks his graph is correct. However, in his graph the relative lengths on the frequency scale have no meaning—liking chicken and chips is in no sense 'more' than liking fish and chips.




**Ian's Graph**




**Kate's Graph**

Kate's graph shows frequencies and it appears that she has just made a trivial error in not spacing the scale marks correctly. However, Kate may be unaware that the relative heights of her columns should be meaningful. While you can 'read' that four like pizza and two like hamburgers, you can't see just by looking that twice as many like pizza as hamburgers.

Their teacher was surprised to realise that Ian and Kate, both capable students, had missed the point of the frequency axis. Often we tell students exactly how to produce each graph or always provide them with pre-drawn and labelled axes. By doing this, we may prevent them from learning to develop the axes and scales for themselves and we may persuade ourselves that they understand more than they do!

# SAMPLE LESSON 1

**Sample Learning Activity:** Beginning—'Colour Graph', page 143

**Key Understanding 1:** We can display data visually; some graphs and plots show how many or how much is in each category or group.

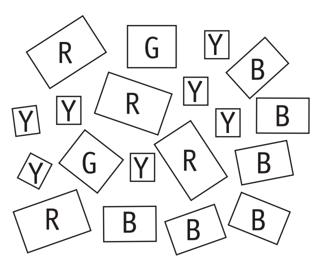
## Teaching Purpose

I wanted my pre-school students to have some early experience in making physical displays of data that they collected in the form of real objects. In particular, I wanted to draw their attention to how they might place their objects to make it easier to see which group has most.

## Action and Reflection

I prepared a bucket of blocks (large red, medium blue, medium green and small yellow) and asked the students to each choose a block of the colour he or she liked best. I asked them to place all their blocks together in the centre of our floor mat so that we could all see the blocks chosen.

When I asked, *I wonder which colour most of us like best?* Jamie replied, *Yellow's the most littlest! The kids all like the littlest yellow ones best. The man makes lots of them for all the little kids.* I explained that we were thinking about only the blocks that we had chosen to help us find out which colour block most of the children in our group liked best. Then I asked, *What could we do to find out which colour most of* ***us*** *like best?*




I realised this was a difficult question for the students. I had to help them understand that we needed to know which colour had the **most** blocks to find out which colour was liked best by **most** people. So I asked: *How could we sort the blocks to help us see which colour has the most?* I was satisfied that the students could see the need to sort the colours when they made four piles—red, yellow, blue and green. However, when I asked, *Can we see which colour has the most now?* Christopher quickly answered, *Red. They're big. Really, really big.* Many of the other students agreed.

I could see that some of the children were using the word 'most' to mean 'biggest'. This is a reasonable meaning, especially since I had asked, *Which colour has the most?* In response, they were attending literally to the amount of redness and that was influenced by the size of the blocks.

*I deliberately chose to use blocks of different sizes as I wanted to draw my students' attention to the numbers of blocks rather than other attributes such as size or length.*

*One student removed her block from the central pile saying:* **That one's mine. I don't want my block in the pile.** *This did not surprise me because I knew young children often want to be able to identify their own piece of data in a display. I needed to persuade these students to consider their blocks as part of the information about the whole group before we could proceed with the lesson.*

However, to answer our original question we needed to attend to the **number** of red blocks, since this was a proxy for the number of people who chose red. I reminded students that we wanted to know what colour most people chose, and rephrased my question: *Which colour has the most **blocks**?*

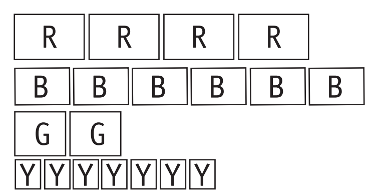
After a pause, I asked, *I wonder if it might help us to see which colour was picked the most if we lined up the blocks?* Liking this idea and eager to help, Simon placed the blocks in one long line, with all the red blocks followed by the blue, yellow and green blocks. This linear arrangement seemed to prompt Darcy to count. He began counting from one end of the line, disregarding the colours, attempting to count all blocks. After counting ten blocks he stumbled over the next few numbers and stopped.

*I would not expect the students to be able to do this independently. I knew I would need to focus my students' attention on the need for a common baseline in a future measurement lesson.*

## Drawing Out the Mathematics

I suggested we could line up the blocks in another way and asked the students to help me arrange the blocks in four separate lines, one for each colour. The children placed the blocks within each line so they were touching and I structured the situation to ensure the lines had a common baseline.

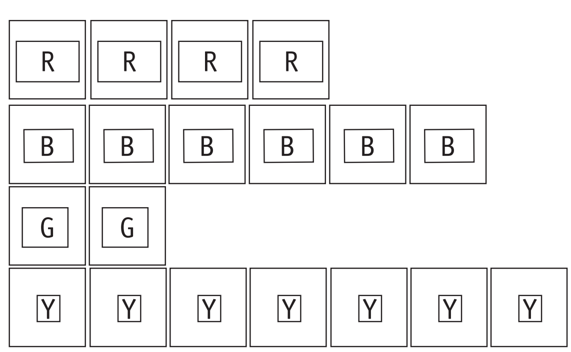
*Can we see which colour has the most now?* I asked.




*Blue!* responded Damien.
*The blue line is the longest but let's think about how many blocks there are,* I said.

*Even though there were fewer blue than yellow blocks, the line of blue blocks was longer due to their larger size.*

To draw the students' attention to number, I placed each block on a piece of identically sized square card and had the children help line up the cards in the same way as before. My intention was to make it easier for the children to compare the collections of each colour, using one-to-one matching, to enable them to see which collection had the largest number.




*I know, it's yellow,* said Georgia. *It's got the most.*
*How could we check to see if yellow has the most blocks?* I asked.

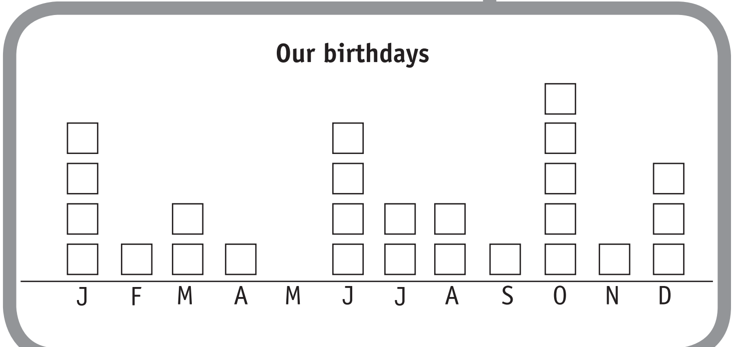
As I'd hoped, this prompted Darcy to want to count again. He carefully counted the blocks one colour at a time, and said. *It is yellow. That's got seven, and blue's only got six, so it is yellow.*

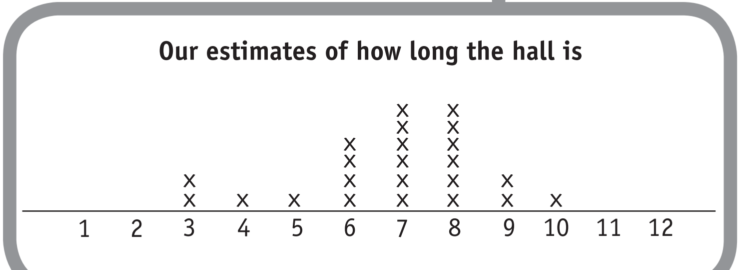
Darcy saw the usefulness of counting as a strategy to solve this problem and had shown the rest of the class. However, I knew that most had not yet developed sufficient understanding of number to recognise when counting is useful, but many could use their emerging ideas about one-to-one matching to see which is more when set out in this way.

### *Did You Know?*

The term 'plot' can refer to graphs that involve plotting each piece of data separately, often, although not always, 'as it comes in'. For example, young students might draw a self-portrait on a sticky note and then stick their pictures above the month in which they were born, thus making a **line plot**. This, in effect, is the beginning of being able to understand **bar graphs**, although line plots are themselves valuable and frequently used tools. Similarly, older students could make an estimate of the length of the hall in metres and each plot her or his estimate directly onto the display. (Sometimes line plots are called dot plots.)

Recording data in this way quickly shows how the attribute of interest is distributed, leading to interpretation and explanation. Recording data 'as it comes in' means that a distribution can change before one's eyes (as when different boxes are opened on election night). This highlights the effect of chance variation on sample data—an important point to be drawn out with students. At times our plot will eventually have all the data in it (population data) so we know what the distribution is, but at other times it will remain a sample of data from the population. Students need to think about how we take this into account when drawing conclusions. (Link to Collect and Organise Data, Key Understanding 5; and Interpret Data, Key Understanding 3.)



## KEY UNDERSTANDING 2

***We can display data visually; some graphs and plots show how one quantity varies over time.***

Plots and graphs showing how a quantity changes with the passage of time are probably the most familiar and readily understood form of two-variable display. For this form of data, we usually use line graphs that are based on using straight lines to join points plotted at discrete intervals. Alternatively, we draw continuous curves that model how we think the quantity changes between the data points we know.

These types of graphs are used when it is meaningful to think about what the frequency or measurement is at a particular moment in time, and to think of it as changing continuously so that it is possible to interpret 'in between' times. For example, it is meaningful to think of and plot the height of a child at any particular time, one's hunger level at any time of the day, or the total distance one has travelled. It does not make sense to think of or try to graph the amount of rain that falls at any particular moment, since to measure rainfall we would have to do so over a span of time. However, it does make sense to think of the cumulative rainfall at any particular time of the day or month or year.

During the middle years, students should begin to plot 'trend' data that is based on their own data collection as it occurs in measurement activities and across the school curriculum. This could include, for example:

- their measurement of the height of the sweet potato plant
- the cumulative rainfall for a three-month period based on data collected daily over the Internet (and perhaps compared with the cumulative 'school' rainfall from their own rainfall gauge)
- the money they have raised to send to a charity
- the distance they have travelled through the day as measured on a pedometer on their belt
- their hunger level estimated on a scale of 1 to 10 at 15-minute intervals throughout the day.

During the later years, students should also begin to make sketch graphs that represent familiar experiences. They could, for example, sketch curves to show how their mood varies through the day or to estimate the noise level in their classroom at different times of the day. The aim is for students to understand that graphs are intended to help us get a feeling for how variables are related to each other.

## Progressing Through Key Understanding 2

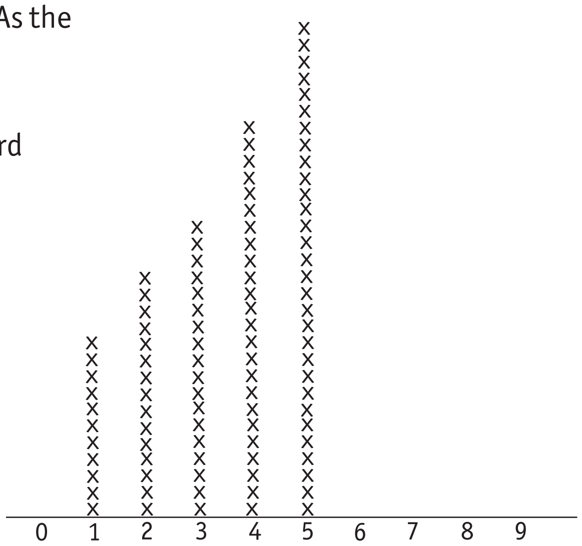
Initially students use the height of bars to represent measurements they have made at equal intervals over a period of time, for example, the height of a sweet pea plant at 10 a.m. each day. As students continue to progress they can produce a graph in which the horizontal axis shows the progression of time and use a vertical scale to plot data points at equidistant times, which are clearly marked on the horizontal axis. They understand when and why they can join the data points to produce a simple line graph. Next, students will be able to produce sketch graphs that 'give a feel for' how familiar things change over time without recourse to careful data collection or point plotting. They can also plot available time data using more complex scales, where each time may not be represented on the horizontal axis.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Collecting Cans (1)

When students are confident in producing simple block graphs and dot plots, have them collectively produce a cumulative dot plot over time. For example: As the class collects cans for recycling, count/calculate the total at the beginning of each week and record as a large class dot plot. (Link to Key Understanding 1.) Focus attention on the top dot or cross in each column and ask: What does the height of these show? Could they go up and down? Why? *(The upward trend shows more and more cans as the weeks go by.)*




### Collecting Cans (2)

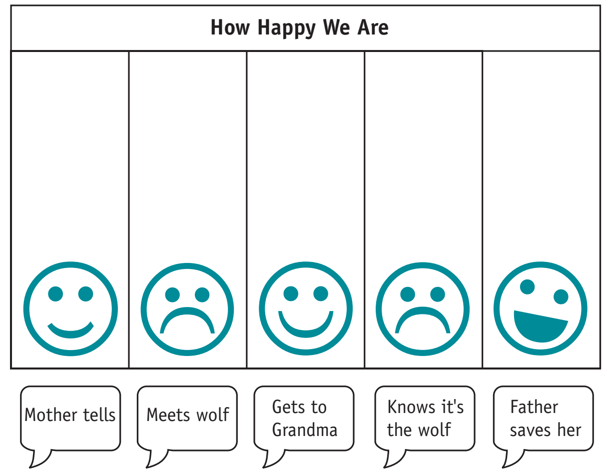
Extend the previous activity by helping students to just plot the top point. For example, at the end of week six, say: As the number of cans increases, it is going to take a long time to put in all the crosses each week. Sometimes to save time, we work out where the top dot would be and just put that in. Highlight in felt pen the top 'dot' in each column. Ask a student to indicate where they think the next top dot will be and plot that. Repeat over successive weeks.

### Growth Measurement

Build up class graphs over time using growth measurements from science and society and environment activities, e.g. the number of marbles that balance each baby guinea pig each week, the height of the sunflower seedling each day. Help students produce bar graphs of their data (link to Key Understanding 1). Use felt pen to highlight the top of each column and ask what it shows. Ask: What do you think would be happening to the guinea pig/sunflower plant between the times we measured it?

## Feelings (1)

Talk to students about easily recognised feelings such as anger, happiness or sadness and what it means to be not angry, a little angry, quite angry, or very angry. Use drawings of faces showing different levels of these emotions for students to rate how happy or angry different incidents in a story make them feel. During a second reading, stop after every incident so students can choose a face and paste it in order across the page.

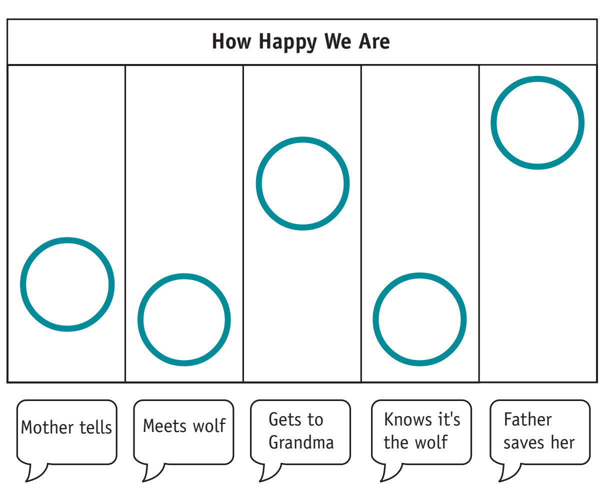



## Feelings (2)

After the previous activity, relate strength of feeling to height, asking students to sit, kneel, stand, or stand very tall, according to the strength of a particular feeling as a story is told. Make sure the focus is on the relative strength of a single emotion, so they know that if it's about feeling sad they stand tall, just as they'd stand tall if it's about feeling happy.

## Feelings (3)

After the previous activity, ask students to think about how they might use the position of objects on a page to indicate different strengths of feeling. Ask: What if we didn't look at the face? If the faces were circles without mouth and eyes, is there a way to tell how you feel by where we put the circles? Refer to the previous activity to draw out that we could put them higher or lower on the page to show more or less feeling, e.g. 'very happy' would be near the top of the page, 'not at all happy' would be low on the page.

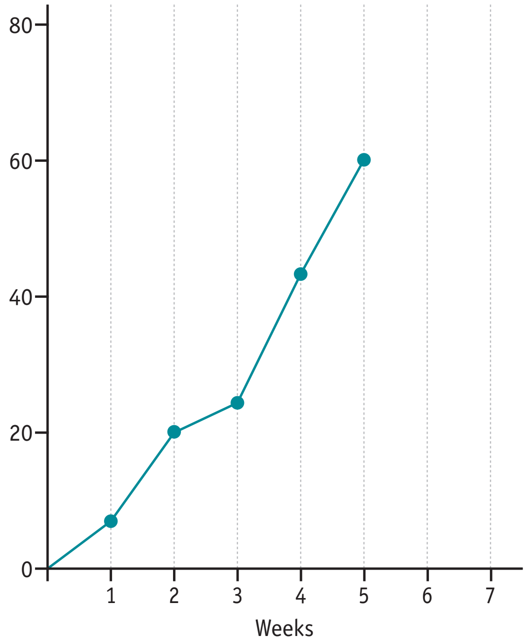


# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Collecting Cans

Extend 'Collecting Cans' (page 160). As a class, plot the-cumulative number of cans collected at regular intervals. Count/calculate the total at the beginning of-each week and show this on a line plot. Mark a single point at the top of each column. Join the points and ask: What might this line show? *(The upward sloping line shows more and more cans as the weeks go by.)* What would a mark on the slope halfway between two of the weeks show? *(If the cans are collected each day, it shows about how many cans we might expect in the middle of the week.)*




### Changing Data

Have students consider data that refers to growth or change over time, e.g. the height of the wheat plant each day, the length of a shadow every half hour, the mass of a melting block of ice every five minutes. Create a class bar graph so that each bar represents the measurement at each time interval. Have large gaps between each of the bars. Ask: Could we put other bars between the ones we've got? What would those bars tell you? How long would we expect them to be?

### Story Map

As a class, to make a story map showing incidents in a story sequentially, then list these in order below the baseline of a graph. Have students decide on a 1 to 10 scale to show how exciting the story is during each of the incidents, and then plot these points using a vertical axis labelled 1 to 10.

### Story Graph

Invite students to individually develop their own 'story' graph from a class story they all know very well, using a scale that involves a strength of feeling. As in the previous activity, have students list the main incidents in a story sequentially below the baseline of a graph. Have different students choose different emotions to plot, e.g. excitement, happiness, boredom or fear. Ask them to think about the numbers on the vertical axis meaning more or less 'strength' of whichever feeling they choose. Compare graphs and discuss. Ask: Why does the excitement graph go down at the same place as the boredom graph goes up?

## Strength of Feelings

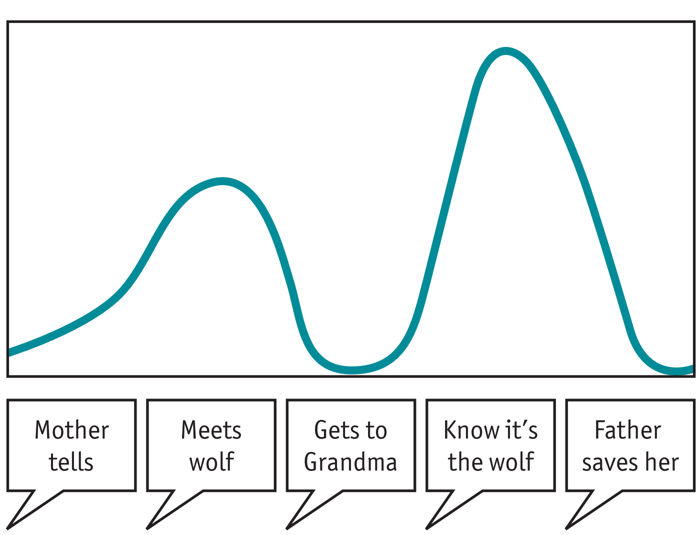
As a class, establish a scale for different feelings, e.g. 'not', 'a little', 'some', 'a lot'. Have students divide up a page horizontally above a baseline, with each section relating to increasing strength of feeling. List story incidents along the baseline, and have students mark how they (or different characters feel) about the incident—higher up the page according to the strength of-feeling.

| | How frightened Red Riding Hood felt | | | | |
|---|---|---|---|---|---|
| a lot | | | | X | |
| some | | X | | | |
| a little | X | | | | |
| not | | | X | | X |
| | Mother tells | Meets wolf | Gets to Grandma | Knows it's the wolf | Father saves her |

Ask: Would it make any sense to join the marks? Why? Why not? Draw out that there may not be a gradual change of feeling—characters might change feelings suddenly, or be feeling different between incidents.

## Up and Down

Tell a short story or incident that stimulates a build-up of excitement or fear. Have students help you to draw a line from one side of a page to the other to represent their feeling as the story progresses—the line slopes up as fear or concern increases, and down as fear or concern decreases. After the story, retrace the line and recall its meaning. Ask: What does the line tell you when it goes up?



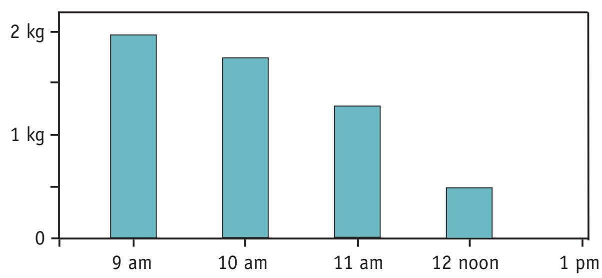
# SAMPLE LEARNING ACTIVITIES

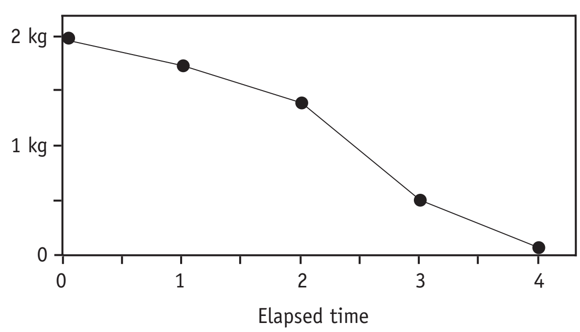
## Later ✔✔✔

### Collecting Cans

Build on 'Collecting Cans' (pages 160 and 162). Have students plan their own axis and scale and keep their own cumulative record as data comes in. They join the weekly points. Ask: Can you estimate about how many cans you will have collected by the end of term if you keep collecting at the same rate? (Link to Interpret Data, Key Understanding 2.)

### Growth and Change




Have students consider straightforward data that refers to growth or change over time, e.g. the height of the wheat plant each day, the length of a shadow every half hour, the mass of a melting large block of ice every hour. Ask students to create a bar graph so that each bar represents the size at the time each measurement was taken. Help them understand that the change or growth continues between the times the measurements are taken, and this can be shown on a graph by making the horizontal axis into a timeline (time scale) and joining up the measurements. What can the height and slope of the line between the measurements tell us?

### Line Graph

Have students create line graphs using data obtained from time intervals of growth, e.g. the length of shadows over a day, the growth of a seed over a week, the mass of a baby guinea pig over several weeks. Ask them to use a time scale on the horizontal axis to plot every second measurement and join the dots. Invite students to use their graph to estimate the missing measurements and compare the actual measurements taken at those times. Ask: Do they match? Why? Why not?

### Height Measurements

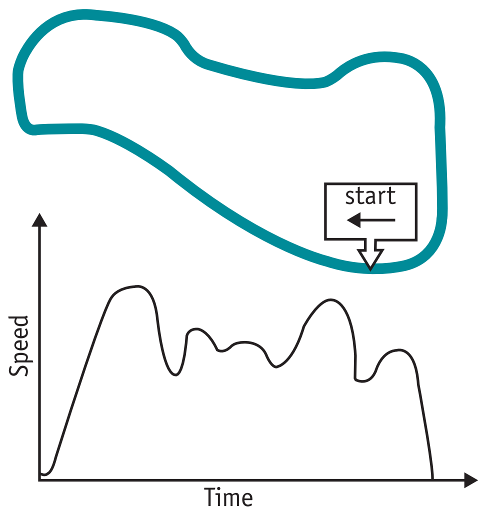
Arrange for students to visit the pre-school or Year 1 classroom each month and take careful height measurements of the children (each student can be partnered with a child whose height they measure each month). Ask students to graph their partner's growth over the year, joining each new growth point with a straight line. Towards the end of the year, ask: What does the slope between the measurements mean? Does the slope change after some of the points? Does this mean the child suddenly changes their growth rate, or would the change be more gradual? Show how joining the points with a curving line is more likely to reflect the child's true growth pattern.

### Informal Line Graphs

Have students sketch informal line graphs to show how various things change over time. They explore the idea of 'more and less' of some attribute in relationships, e.g. the height of water in different-shaped jugs as they are filled from a steady flow; the excitement level in a story as it is told; the circumference of a balloon as it is blown up. Focus on what increases or decreases, when there is more and when there is less of it, and when the quantity may stay the same.

### Racing

Extend the previous activity by asking students to make a sketch graph representing the speed of a toy electronic car travelling around a circuit. Have one student call out every ten seconds 'very slow', 'slow', 'medium', 'quite fast', or 'very fast', as they all watch the car slow down around corners and speed up on the straights. They record the calls. Have students use this data to sketch a line graph of the speed of the car over time. Compare the graph with the shape of the track. Invite students to draw different circuits, then sketch speed/time graphs that match the circuits. Mix and distribute several circuits and graphs to different groups and then have them come to a consensus about which graph matches which track. (Link to Interpret Data, Key Understanding 2).




### Feelings (1)

Read or tell a short story that stimulates a build-up of tension or excitement and have students draw a line from one side of their page to the other to represent the strength of this tension or excitement as the story progresses—the line slopes up as tension increases and down as tension decreases. At the end of the story, ask students to retrace their line and recall its meaning.

### Feelings (2)

Extend the previous activity by having students construct graphs of the build-up of tension or excitement over time in short stories they read using the same tension scale on the vertical axis. Ask: Are there typical patterns in the graphs for different types of stories?

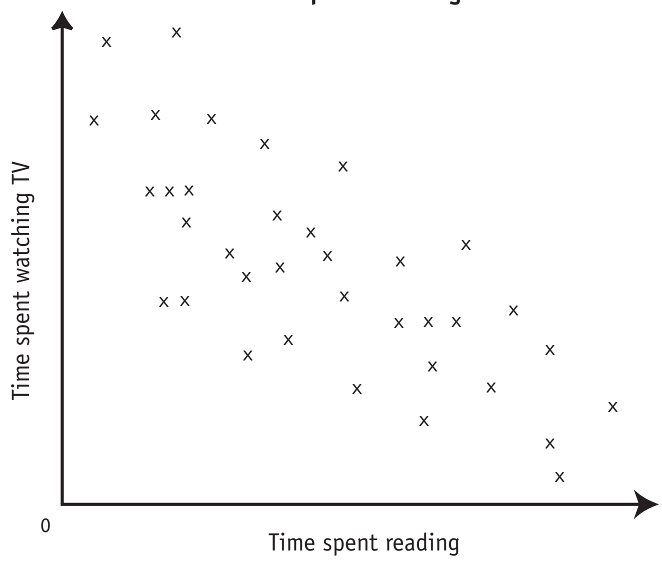
## KEY UNDERSTANDING 3

### *We can display data visually; some graphs and plots show how two quantities are related.*

Often we wish to display data in a way that shows us whether or how two variable quantities are related. In order to be able to display data involving two variables, students need to be able to coordinate the information on two axes at once. Games that involve placing objects in specific squares in an array or using coordinates to locate things can help with this process, and can begin during the middle primary years. There are many computer games that help students to develop this skill.

Students should begin to use plots and graphs involving two quantities during the later primary years. For example, to investigate whether there is a relationship between how much TV children watch and how much they read, students could collect information from a sample of children of the hours of time spent in each of the two activities each week. They could then make a graph by labelling one axis with the variable 'TV time' and the other with the variable 'reading time' and plot an ordered pair (TV time, reading time) for each child. The resulting scatter plot would then give some idea about the extent and nature of any relationship. (Link to Interpret Data, Key Understanding.)

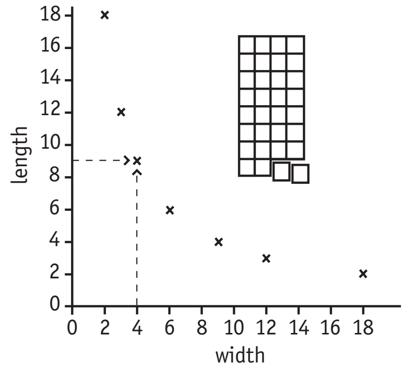
**Relationship between time spent reading and time spent watching TV**

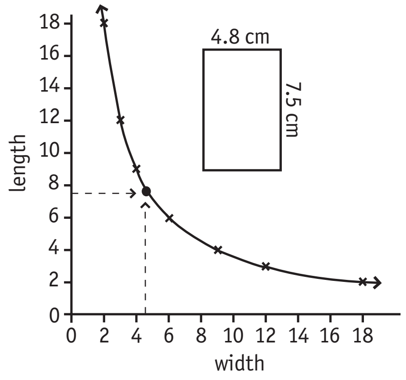


Students should also use graphs to investigate measurement relationships. They might measure various circular lids to find their diameter and circumference, or the height and head circumference of students in their class. In each case, they can graph the pairs of values obtained. When plotting the circumference of circular lids, the points should, theoretically, lie on a line but the chance variation introduced through measurement means that the points are likely to lie close to, rather than exactly on the line. As long as the measurement is reasonably precise, the underlying relationship will still be evident and students should be able to predict the circumferences of other lids. (See Interpret Data, Key Understanding 2.)

Sometimes one variable can be predicted from another. For example, when a student, after playing 'Guess my rule', says that the rule is *double the number and add one*, he or she is describing a quite precise relationship between the input number and the output number: output number = 2 × input number + 1. In this case, plotting the ordered pairs (input, output) will show that all the ordered pairs are on a line, enabling the prediction of other pairs of values.

Students should learn to decide whether it makes sense to join the points on their graphs, and to explain their decisions. For example, they know that if they were plotting pairs of points that showed the-length and breadth of rectangles composed of 36 centimetre squares, only whole number values would be sensible on the axes and joining points would not make sense. If they were plotting pairs of points that showed the length and breadth of *any* rectangle of area 36 square centimetres, any numbers would be sensible on the axes and joining points of the graph would make sense.




Eventually students can represent two-variable data in scatter plots and simple coordinate graphs.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Ordered Measurements

Extend 'Measurements' (page 144) by repeating the activity for containers that go up in size systematically. For example: Collect soft drink containers clearly labelled as one-, two- and three-litre sizes. Have students measure and record how many cupfuls fit into each bottle. They then produce a block graph on squared paper using the container size as categories. Encourage them to put the containers in order on the horizontal axis and equi-spaced. What happens to the columns as the container size goes from 1 to 2 to 3?

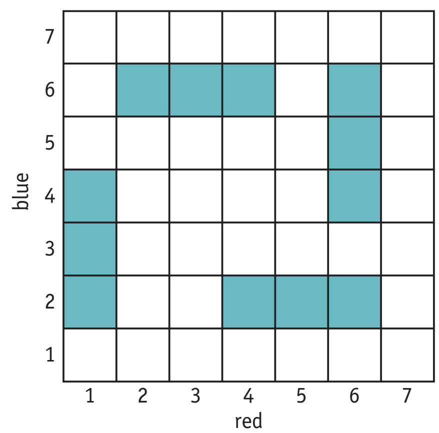

### Two-way Grids

In the context of various games and activities, encourage students to begin to place things in a two-way grid. This is the beginning of being able to plot points, although students would generally not be expected to hold two variables in their mind at once. (Link to ***First Steps in Mathematics: Space***, Location, Key Understanding 1.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔

### Battleships




Have students play games where they need to coordinate two axes at once. For example, have them use numbers to-mark grid references on squared paper, distinguishing vertical columns and horizontal rows by colours (see diagram), labelling the spaces between the grid lines. They then secretly place four battleships onto their grids by choosing three adjacent squares for each ship. Players have three turns to 'hit' their partner's ships by calling out grid references (e.g. red 3, blue 2). Two 'hits' are required before a ship is sunk. (See ***First Steps in Mathematics: Space***, Represents Spatial Ideas, Part B, Key Understanding 1.)

### Physical Grid

Set up a physical grid on the pavement (with chalk, rope or based on square pavers). Along the bottom of the grid write 'height', and on the side write 'weight'. Show pictures of a variety of animals of varying proportions. Ask students which part of the grid would a snake go into? What about a giraffe, an elephant, and an emu?

### Physical Scatter Plot

Set up a physical scatter plot using ropes or the edges of the classroom as the two axes. Label one axis 'not very hungry to very hungry' and the other axis 'not very tired to very tired'. Ask students to position themselves along the horizontal axis according to how hungry they are, and then walk forwards until they have reached the appropriate point on the vertical axis to show how tired they are. Ask: Where should you be if you are very hungry, but not very tired? Where would you be if you are not very hungry, but very tired? What about if you are very hungry and very tired? Draw out what the different parts of the grid mean.

### Coordinate Grid

Following the two previous activities, set up a coordinate grid on the display board, and use it with the students to explore a variety of familiar relationships. Have students make a labelled pin with their name on it, and use it to show their position on the grid. Encourage students to contribute ideas for the grid, e.g. going to bed time and getting up time, liking sport and liking reading. Ask: If you like both sport and reading a lot, where will your pin go? If you like reading a lot but not sport, where would you put your pin?

# SAMPLE LEARNING ACTIVITIES

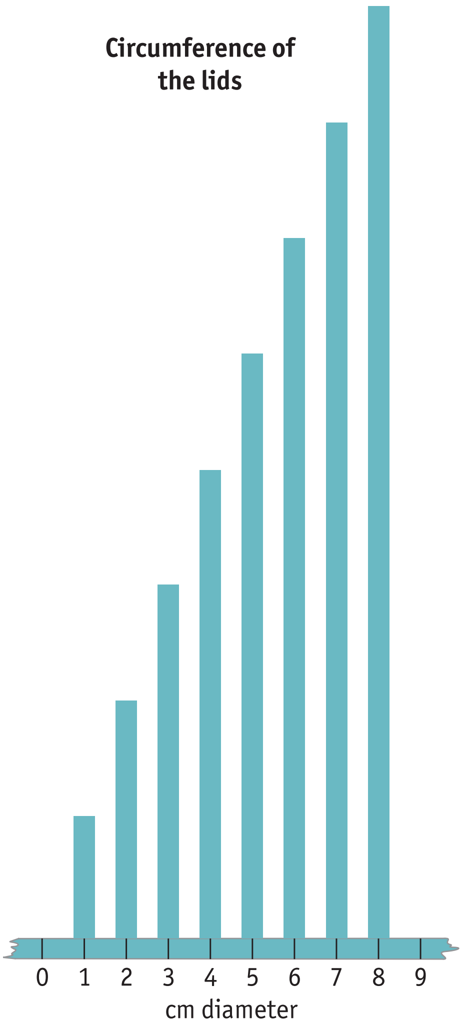
## Later ✔✔

### Skeletons

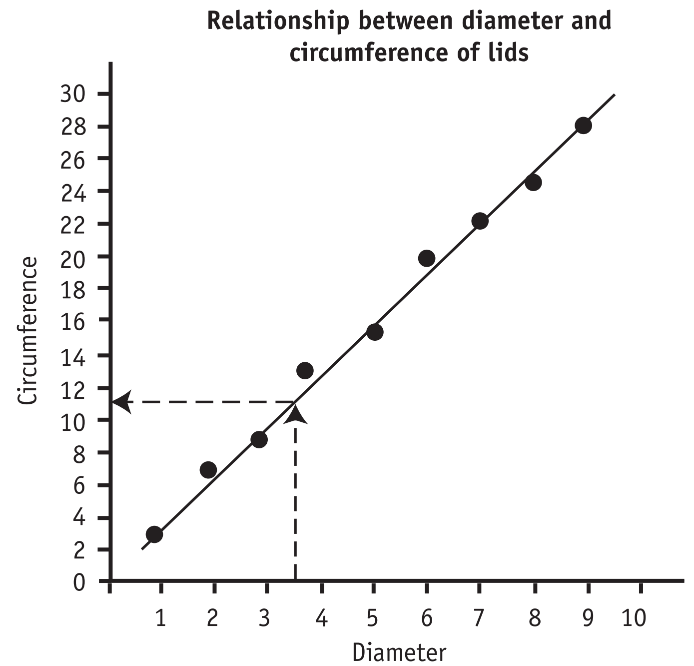
When human bones are found, forensic specialists can estimate the height of the person from the length of a leg bone. Ask students to produce two pieces of measurement data on themselves: their height and the length from their ankle to knee. Set up a class scatter plot using a large sheet of one-centimetre grid paper on a pin-up board, labelling the axes in two-centimetre increments on each grid line. Label the horizontal 'ankle to knee' and the vertical 'height'. Have students place a pin on the plot at the appropriate grid position for their two measurements. Encourage them to be systematic in finding the appropriate location, first using the horizontal axis and then up to match the vertical value. Ask: Are there patterns in the placement of the pins? Do you think ankle-to-knee length is related to height? Do the taller people generally have longer legs from the ankle to knee? Do the shorter people generally have shorter legs? (Link to Interpret Data, Key Understanding 2.)

### Measuring Lids (1)

Have each student measure the diameter of a different sized lid to the nearest centimetre and then cut a piece of tape the length of the circumference. Create a class display of the data by attaching a paper strip with a centimetre scale from zero to beyond the longest diameter across the bottom of a pin-up board. Have students attach their circumference paper strips vertically above the respective diameter length. Ask: What can you say about the 'graph' we've created? Why do the circumference lengths go from shortest to longest along the diameter strip?



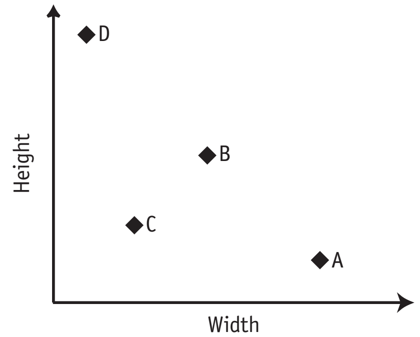
### Measuring Lids (2)

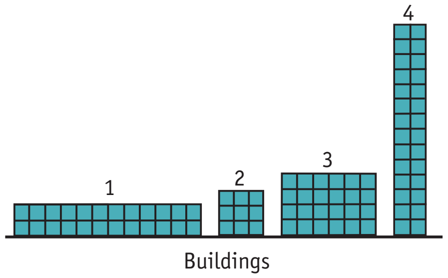



Extend the previous activity to produce a scatter plot. Have students draw a pair of axes on centimetre grid paper and label the horizontal axis 'diameter' (and number the centimetre grid lines by ones) and the vertical axis 'circumference' (and number the centimetre grid lines by twos). Invite them to plot points on their coordinate grid to represent data from the previous activity. Draw out that each point represents a lid, and provides two pieces of information about the lid. Ask: How is your scatter plot similar to the paper tape display? Would it make sense to draw a line from the lowest point to the highest point on both displays? Why aren't our points all exactly on the line? (*Our measurements of the lids are not accurate enough; they were rounded up or down to the nearest centimetre.*) If we measured a lid with a diameter of, say, 3.5 cm, could we use the line to approximate the circumference? Why is the slope of the diagonal less on our scatter plots than on the wall display? (*We used different scales on the axes.*)

### Buildings

Have students look at a picture of four 'buildings', represented by rectangles of different proportions and try to match them to points marked on a simple coordinate graph. Explain that the horizontal axis shows how wide the building is, and the vertical axis shows how tall it is.




### People Graph

Provide students with pictures of four people—two old, one tall and one short, and two young, one tall and one short (see Interpret Data, Key Understanding 1) and have them informally plot the points on a provided pair of axes.

## Later ✔✔

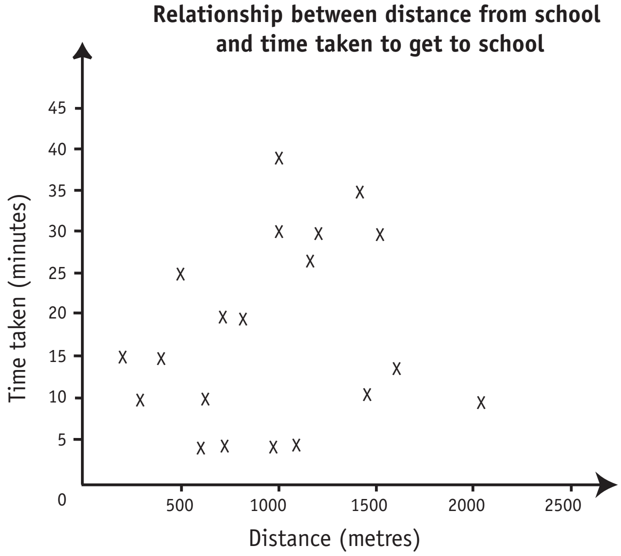
### Ordered Measurements

Have pairs of students collect different-shaped containers for which they know the capacity, e.g. soft drink bottles, milk or juice cartons, measuring cylinders and cones. Ask them to fill each container with water and weigh the water (weigh the empty container and full container and find the difference, or pour into a scale). Have one student in each pair draw a bar graph, one showing the mass, and the other showing the capacity of their containers. Have them compare their graphs. Ask: Is there a relationship between mass and capacity? How do you know? Then invite each pair to construct a scatter plot, combining the two types of data. Ask: Is it easier now to see the relationship between the two measures? Why? Would a line joining the lowest to the highest point make sense in this relationship? (*Yes, because there is a direct relationship between the millilitres and grams of water, 1 mL = 1 g water.*) Should any of the points fall outside the diagonal line? (*No*) Can you say what the mass of water would be for other containers of known capacity? (Link to ***First Steps in Mathematics: Measurement***, Understanding Units, Key Understanding 8.)

### Exploring Relationships

Have students brainstorm a wide range of data types, e.g. weight, height, length, thickness, distance, time spent, scores, various lengths, ratings of attitudes and feelings, ratings of physical characteristics of objects such as hardness, strength and absorbency and so on. Invite students to consider and predict various relationships that they can later explore by taking two kinds of measurements and producing scatter plots. For example: They might explore height and weight data for Olympic athletes and make a scatter plot for each different event to explore the relationships involved. They might create a scatter plot to see if there is a relationship between the distance they live from the school and the length of time taken to get to school. Ask: What does your scatter plot look like? Are the points spread out all over the grid? (*No relationship*) Do they seem to cluster about an imaginary diagonal line? (*Some kind of relationship*) What can you say about the relationship between the data when the points slope up from left to right? (*When there's more of one measure, there's also more of the other*) What if the points slope down from left to right? (*When there's more of one measure, there's less of the other*) What if the points seem to cluster in a bunch in one place on the plot? (*There is not much variation in the data, or the scale on the axes cover too much range*) Can you explain why the points are so spread out on the scatter plot for distance

**Relationship between distance from school and time taken to get to school**

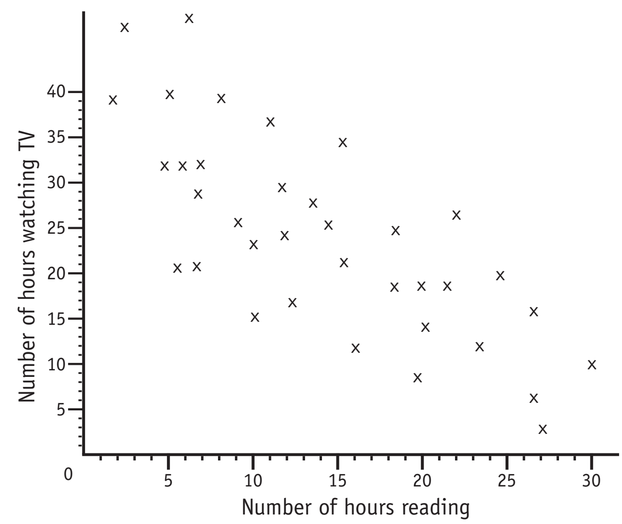


and time taken to get to school? Why don't those living furthest from the school take the longest to get here?

### Time Relationships

Have students explore time relationships by making scatter plots and examining the clusters. For example: Collect data over a week about how much time students spend watching TV and reading. Draw a graph, labelling one axis 'TV time' and the other axis 'reading time' and plot the ordered pair (time watching TV, time reading each week) for each student. Ask: Can you say just by looking whether or not there is a relationship? Ask: Would it make any difference to the look of the graph if the axes were reversed? Why? (Link to Key Understanding 2.)

**Relationship between time spent reading and time spent watching TV**

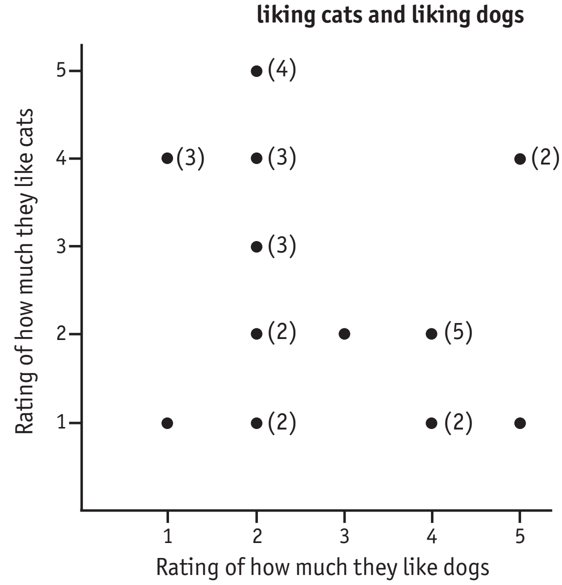



### Ratings Data

Have students construct scales to record strength of feelings for two different things. They collect data from others and produce a scatter plot to help them look for relationships to test their predictions. For example: They could ask their classmates to rate their degree of liking for dogs and cats separately on a five-point scale to test their prediction that cat lovers don't like dogs and dog lovers don't like cats. Ask: How will you label the axes? Does it matter if two or more points fall on the same spot? Draw out the need to distinguish between one piece of data at a point and a number of pieces of data at the same point. (Model the convention of showing a bracketed number next to the point when more than one piece of data is represented by one dot or cross.)

| Tick the box. | hate them<br>1 | don't like them much<br>2 | they're OK<br>3 | like them quite a lot<br>4 | love them<br>5 |
|---|---|---|---|---|---|
| How much do you like cats? | ☐ | ☐ | ☐ | ☐ | ☐ |
| How much do you like dogs? | ☐ | ☐ | ☐ | ☐ | ☐ |

**Relationship between liking cats and liking dogs**



# KEY UNDERSTANDING 4

## *We use tables and diagrams to organise and summarise data in a systematic way.*

Much of the information we collect or respond to comes in an unstructured way. We record the colour of cars as they arrive at the intersection, the numbers on the die as they appear and the answers to the questions as people give them. Lists, tables and diagrams enable us to organise data to enhance its accessibility and meaningfulness. Sometimes we plan ahead and use pre-formatted tables and diagrams to organise information as it 'comes in', perhaps producing a tally, or recording information directly into a computer database. Sometimes we organise it later. For example, we may collect information about different holiday options and then sort this information into a table to help us make comparisons.

Tables are used to exhibit data in a definite and compact form or scheme where the arrangement of the information is significant in interpreting it. The simplest table is a list, which may or may not be-ordered. An alphabetical list of names and the 'six times table' are examples of organised lists. Slightly more complex are one-way tables that require students to place information in the right position in relation to other information. As suggested in Interpret Data, Key Understanding 1, this structure is not always immediately obvious to students and it needs explicit attention.

| Name | Eye colour |
|---|---|
| Mai | blue |
| Freya | brown |
| Peter | blue |
| Kim | brown |

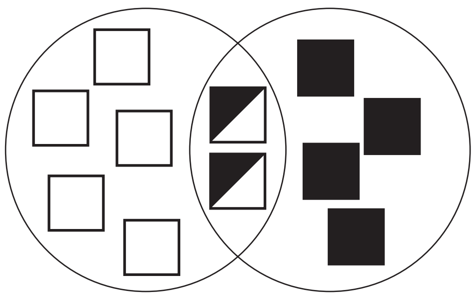
| Eye colour | Frequency |
|---|---|
| blue | //// |
| brown | // |
| green | / |

Two-way tables are more complex and require students to coordinate two constraints at once. Using the table at the top left of page 175 requires them to understand the grid cell structure and find the appropriate intersections of columns and rows. To produce the right-hand table, they need also to understand how frequencies in rows and columns may be summarised to show totals, often without the word 'total' being used explicitly or the column or row being labelled.

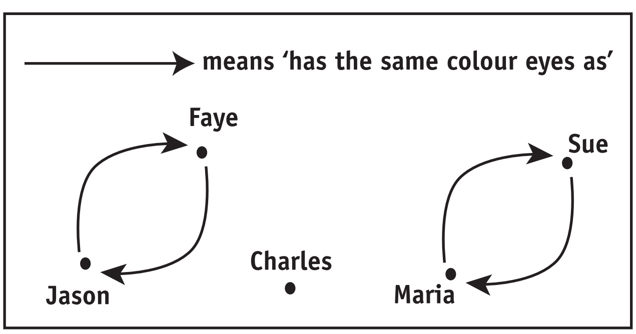
| | Heads | Tails |
|---|---|---|
| Heads | HH | HT |
| Tails | TH | TT |

| | Girls | Boys | |
|---|---|---|---|
| Year 6 | 15 | 13 | 28 |
| Year 7 | 12 | 18 | 30 |
| | 27 | 31 | 58 |

Other diagrams such as Venn diagrams and arrow diagrams (see below) may highlight relationships between categories or things.



**Venn diagram**




**Arrow diagram**

Students should organise data in a range of lists, tables and diagrams with specific attention being drawn to the way the structure of these representation methods helps us to organise information.

## Progressing Through Key Understanding 4

Initially students can use organised lists and one-way tables to organise information. As students continue to progress further they can use Venn diagrams involving two overlapping categories and can place information into the correct location in simple two-way tables. Next, students fluently use Venn diagrams and two-way tables and can also construct arrow diagrams. As students progress further they can display information in tables involving provided class intervals.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔

### Equipment

Have students draw the individual pieces of play equipment stored in the outdoor shed and involve them in making a representation of how many of each item of equipment is stored (e.g. draw two tractors, four shovels, seven skipping ropes). Help students to set their drawings out in a table. They then use the table to help them check whether any items are missing after packing away equipment.

### Modelling

Model the process of producing simple lists and tables incidentally in school activities. For example: When students are reporting on their progress on a task, produce a table with labelled columns '(Haven't Started', 'Written Story', 'Collected Pictures', 'Finished'). As students report their progress, record their name on the table. After the first day, have students rub their name off the table when their progress changes and record their name in the appropriate part of the table.

### Language Groups (1)

Ask students to collect data about their classmates and decide how to organise their information. For example: After they have recorded the different languages spoken in the class, ask: How could we organise the information so we can quickly find all of the students who speak Chinese? Draw out that lists and tables help us find information quickly if organised in a systematic way.

### Language Groups (2)

Extend the previous activity to include data where there are likely to be students who belong in more than one group. For example: How can we show the students who speak more than one language? Could we write their names on both lists?

### Food Groups

During health lessons, ask students to create a list of the different food types and add items to each list whenever they eat one of the foods. Ask: How does the list help you see what you have eaten most of? Could you organise the information in a different way to help you see things at a glance?

## Where Do You Fit?

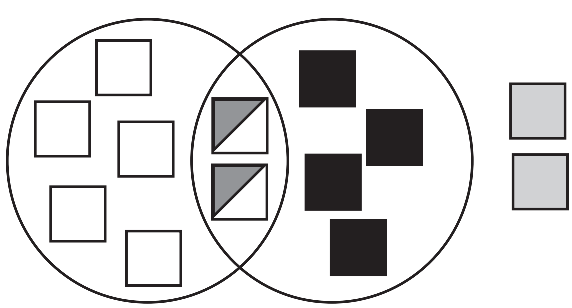
Divide the classroom in half lengthways by running a piece of paper tape down the centre (see diagram below). Label each half of the room so that all students fit into one category or the other (e.g. boy/girl, wearing sneakers/not wearing sneakers, brought their lunch with them/purchased lunch from the canteen). Invite students to decide where to stand. Ask: What made you decide to stand on that side of the room? What do we know about Claire if she is standing over here?

| Canteen lunch | Lunch from home |
|---|---|
| | |

Ask: How could we record this information to show someone what we have found out? Draw a table on the board to represent the classroom, and list the students who stood in each section. Ask: Where was Tom standing? Where will we write his name? Draw out that the order of the names is not important (i.e. if Tom was at the front of the classroom he doesn't need to be at the top of the grid), but putting them in the right section is.

## Venn Diagrams

Have students use two hoops to sort categories of data that overlap. For example: After reading a scary story to the students, ask them to find out what scares most children in their class. They draw what it is that scares them and then use two hoops to show two categories of 'scary things'. Show how overlapping the hoops allows a new space for pictures that show both scary things. Ask: Where would we put pictures that are about neither of those things? (Link to Collect and Organise Data, Key Understanding 3.)



# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Modelling

Model the process of producing organised lists and tables incidentally in school activities. For example: Produce a proforma for groups to use when working out a roster for sharing sports equipment. Have students enter relevant data for their own group.

| Name | Monday | Tuesday | Wednesday |
|---|---|---|---|
| Fiona | football | hoops | tennis gear |
| Paul | hoops | tennis gear | football |
| Jade | tennis gear | football | hoops |

### Favourite Fast Foods

As you help students to plan their survey data collection, model the process of producing an appropriate data collection table by 'talking aloud' as you sketch a table on the board. For example:

| Our Favourite Fast Food—Room 5 | | |
|---|---|---|
| **Food** | **Tally** | **Frequency** |
| Pizza | | |
| Chinese | | |
| Hamburgers | | |
| Chicken | | |

Provide pairs of students with a copy so that they can record data appropriately as they produce it.

### Food Preferences

Have students compare alternatives when producing tables for data collection. For example: Compare the following possibilities for collecting and/or displaying data on food preferences. (Link to Collect and Organise Data.)

| Name | Food liked |
|---|---|
| Karen | Pizza, hot dog, Hungry Jack's |
| Aaron | Hot dog, Hungry Jack's, KFC |

| Food liked | Name |
|---|---|
| Pizza | Karen, Lisa, Ian, Tom, Cassandra, Bill, Sam |
| Hot Dog | Karen, Aaron, Petros, Lisa, Ian, Cassandra |

| Food liked | Tally |
|---|---|
| Pizza | 𝍸 \|\| |
| Hot Dog | 𝍸 \| |

## Where Do You Fit? (1)

Extend 'Where Do You Fit?' (page 177). Introduce a second pair of categories and have children re-sort themselves. For example: After students have sorted themselves into 'lunch from home' and 'canteen lunch' groups, divide the room in half again at right angles to the first and label the rows 'had a drink with lunch' and 'didn't have a drink' (see diagram below). Ask students to decide which square they belong in now. Record the students' names in each section on a two-way table drawn on the whiteboard.

| | Canteen lunch | Lunch from home |
|---|---|---|
| **Had a drink with lunch** | | |
| **Didn't have a drink** | | |

## Where Do You Fit? (2)

Have students produce their own version of the table in the previous activity. Working in pairs, one student reads the names in one cell from the table on the board and the other produces a tally. Change over for next cell. Ask students to produce a final table of data using the frequencies from their tally. Help them to sum up the columns and rows and explain what they show.

## Useful Tables

Ask students to examine a range of tables containing information ordered in different ways, e.g. sporting team player lists, names by alphabet, frequency of lotto numbers by size. Ask them to answer questions such as: 'Which player is number 14?' or 'How often did number 15 come up?' Ask: What makes it easy to find the information you want? Sally, you found the answer to that question very quickly—what helped you? Did you have to read the whole table, or did you know where to look to find the information? Ask some questions that are less well facilitated by the table, e.g. which number a certain player wears. Ask: Did the table help? How could you rearrange the information to make it easier to find? Draw out that useful tables are ordered by the information people are most likely to want.

## Venn Diagrams

Extend 'Venn Diagrams' (page 177) by having students draw and label the circles. Have them count how many are in each category and write the numbers in the appropriate section. Ask: Why don't we need to label the overlapping sections?

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Flight Information

During technology and enterprise activities, have students produce tables to record information about their designs. For example: Have students make planes out of different materials and test them to see which flies the furthest. Ask: What are some choices in the way a table can be organised? Where will you write the name of the plane? How many columns will you need to allow for your information? What will you label the columns? Is there a particular order that is better than another? Draw out that labelling such columns in the same order as they will be used is sensible.

### Studying Tables

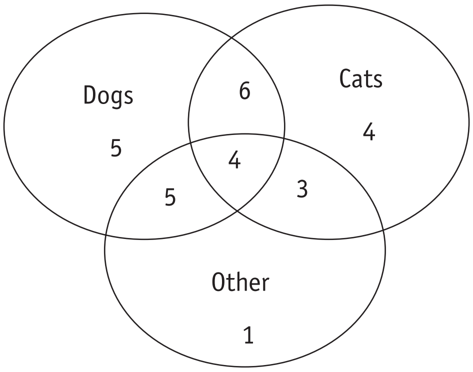
Collect a range of tables from publications for students to study. Draw out features that are common and those that are different. Ask students to copy different table forms and insert their own data obtained from science, environment and society, health, or chance investigations. Ask: Do the tables show totals? How are they indicated on the tables?

### Changing Headings

When students are constructing or analysing tables, invite them to try swapping the headings of the columns and rows. Ask: Is the information still correct? How does changing the headings change the ease of reading the tables?

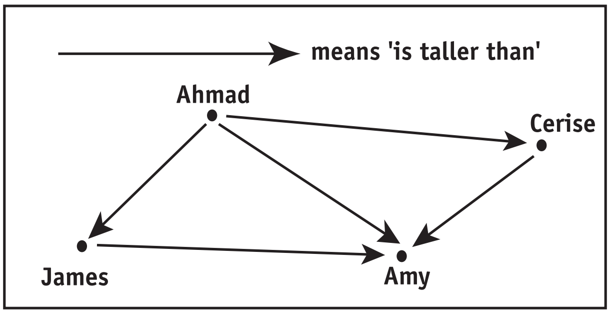
### Venn Diagrams

Extend 'Venn Diagrams' (page 179) and have students produce their own three-circle diagrams. Ask them to collect data, e.g. on pet ownership, and plan what each circle represents. They first use sticky notes with names to put the data into the correct part. They then count how many and label each section. Ask: What do the overlaps mean? How can you check that you've included everyone? Why don't you need to label the overlapping parts?



### Arrow Diagrams

Invite students to represent relationships using arrow diagrams, e.g. to represent relationships between people in their family, or various physical characteristics.




### Maps and Pictures

Explore a range of tables and charts that organise information in different ways, such as populations shown on maps, or pictures on which information is listed. Have students try organising their own data using these techniques. Ask: What use are the pictures or maps? How do they convey information differently to an ordinary table? For example, showing population for each state on a map of Australia means you can find the information quickly without needing to look through a list of places.

### Totals Tables

Ask students to decide whether or not their two-way tables need a total column and/or row. Draw out that it depends on the purpose of the table—would we be interested in, e.g. the total height, weight or age of a group of students?

| Name | Height (cm) | Weight (kg) | Age (yrs) |
|---|---|---|---|
| Karen | 102 | 23 | 8 |
| Jemma | 112 | 29 | 9 |
| Colin | 98 | 22 | 7 |
| Ahmad | 106 | 26 | 8 |
| Maria | 110 | 27 | 9 |

Would we be interested in the total spent in a month by a group of students?

| Name | Books ($) | Snacks ($) | Movies ($) | Total ($) |
|---|---|---|---|---|
| Karen | 5 | 8 | 8 | 21 |
| Jemma | 10 | 10 | 16 | 36 |
| Colin | 15 | 12 | 0 | 27 |
| Ahmad | 5 | 4 | 16 | 25 |
| Maria | 20 | 15 | 10 | 45 |
| **Totals ($)** | **55** | **49** | **50** | **154** |

# KEY UNDERSTANDING 5

***How we display our data depends on the kind of data we have and our purpose.***

As students extend their repertoire and experience with activities for Key Understandings 1 to 4, they should be helped to develop the capacity to make conscious decisions about how to represent data. Students should learn that graphs, tables and diagrams are not an end in themselves, nor are they simply appealing or 'artistic' presentations of data. Rather they serve quite specific purposes in enabling us to more readily understand, analyse and interpret our data and to communicate it to others. Thus, our decisions about how to represent data should take into account the type of data, the messages to be conveyed, and the context and audience for the display.

## The Type of Data

As described in Key Understandings 1 to 3, different types of data require different data handling and display techniques. For example, if we plotted the height of a particular child on his or her birthday each year, it would make sense to join the points to form a line graph, since the line consists of points that indicate the approximate height of the child between years. However, if we selected a number of individual children at each age and plotted their heights, it would make no sense to draw lines connecting the data points, as these lines would have no meaning. Students should be helped to choose methods of presentation that suit the type of data they have.

## The Message to Be Conveyed

Different forms of representations (tables, graphs, plots) will highlight different aspects of the data and they should be chosen with care so as to give a quick, lasting and accurate impression of the significant information. The particular type of table, graph or plot needed will also depend on the aspects of the data to be illustrated or highlighted. Line graphs, for example, are useful for tracking trends over time, while pie charts make it easy to see relative proportions. When producing bar graphs (or histograms), different groupings of data, and even the order in which groups are shown, will convey different messages.

As students perceive the need for increasingly sophisticated forms of data representation, the teacher can help them by introducing new methods of representation. Little is likely to be achieved by providing a collection of data out of context and having students practise drawing graph types in isolation. Students should compare the same data displayed on different types of graphs and with different groupings in order to understand the impact on the messages conveyed and the potential of different displays to mislead. Computer graphing programs free students from the tedious and time-consuming chore of producing graphs manually, allowing them to produce different graphs of the same data and make rapid comparisons between them.

## The Context and Audience

Whether intended for personal use or to communicate information to others, data displays are intended to enhance understanding and communication. At times, all that is required to clarify or make a point is a quick sketch graph. Under such circumstances, fussing with accurate plotting and labelling is a waste of time. However, at other times, accurate plotting and attention to detail may be needed for differences and patterns to become evident. While there are some conventions associated with different types of graphs, such decisions need to be made by thoughtfully taking context and audience into account, rather than according to sets of rules about the 'correct' way to produce them.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Lost Teeth

When students have collected data about those in the class who have lost teeth, ask them to create their own display of the information. Have students compare their displays and say what information they can get from each other's. Ask: Which display helps you to see who has lost the most teeth? Which tells you how many teeth the boys in our class have lost? Ask students how they would change their display to show the different things.

### Favourite Sports

After students have physically arranged themselves or objects to make a display like a bar graph, ask a question that encourages them to represent a back-to-back graph. For example: Ask students to line up according to which sport they prefer. Ask: I wonder if more boys prefer cricket than girls? How can we tell? The boys and girls in our groups are all mixed up. Draw out that the different question made it necessary to use a different type of representation.

### Matching Collections

Invite students to select a handful of different coloured beans from a jar. Ask them to sort and represent their collection on paper to make it easy for someone else to re-create an identical collection. The paper is then given to a partner who attempts to make the collection. Ask: Did your partner's collection match yours? How could you make it easier for someone to go and get the same collection?

### Measuring Height

Have students cut strips of paper tape to match their height. They then write their names on one side of the strip of paper, and display the strips without these names showing. Ask: We know this is the smallest strip, but how do we know who it belongs to? If we were out at sport and someone else came into our room, how would they know who is the shortest? What would we need to do? Draw out the importance of labelling the data when you are communicating with others.

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Fitness Facts

Have students brainstorm types of data they could collect during daily fitness sessions to find out whether their fitness is improving. After they have collected their information over a period of weeks, ask them to suggest ways to represent the information to find out if they have improved.

### Book Data

Ask students to sort the classroom books, then to make a chart to display near the bookshelf. Ask: What kind of data about our books would be useful to display? What would be the best way to present this information?

### Zero Values

Invite students to make decisions about how to represent data values of zero. For example: If they have discovered that no students in the class have green eyes, ask: Should we still include a column for green eyes on our graph? What would happen if we didn't include green eyes? Draw out that whether or not we include zero values depends on the question we want to answer.

### Newspaper Graphs

Have students collect graphs used in the newspaper and compare the differences and similarities between them. Ask: Why do you think the writer chose to represent the data using a line graph? Could we use a bar graph to show the same information? Which would be better? Why?

### Graphs and Tables

Ask students to compare the use of a table and a graph when representing data they have collected. After students have looked at the points scored by each faction/house or house at the sports carnival, ask: What would be the best way to represent this data if we wanted to see quickly how many points blue faction scored? How would we represent the information if we wanted to see quickly which team scored the most? Draw out that tables are often useful for finding specific information quickly, and graphs are often useful for making comparisons and examining relationships.

### Footy Tipping

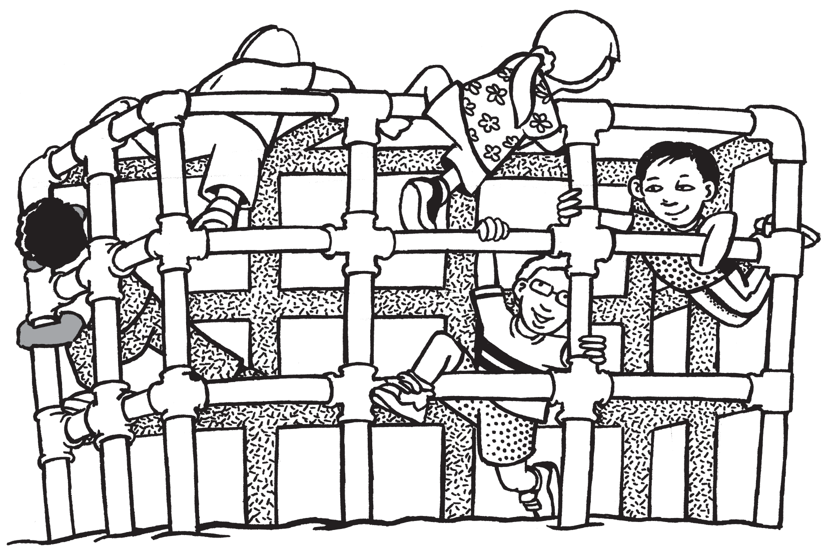
In a class football tipping competition, ask students to decide which information about teams' previous performances might help them make their predictions. Ask: How could we represent the information about previous performance to help us pick our teams for next week? (See Understand Chance, Key Understandings 1, 3 and 7.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Play Equipment

Ask students to consider the purpose of their data display and the message they are trying to convey when making decisions about how to represent their data. For example: After collecting data about which piece of play equipment is the most popular, ask students to decide how to present this data to the school council to convince them to buy more of the equipment they need.



### Class Results

Encourage students to use simple plots as a direct means of exploring some data they have collected, rather than constructing formal tables and graphs. From a class list of scores in a game or test, draw a baseline and label it with the range of scores, then put a cross for each score above its spot, building up a profile of the class results. Ask: In what ways is this helpful?

### Choc-chip Cookies

Have students use real data to investigate the usefulness of dot plots for exploring emerging patterns in data. For example: Have them count the chocolate chips in choc chip cookies, adding their data for each cookie to a class line plot as they finish counting the chips (and eating) each cookie. Ask: How does the plot change as each new lot of data is added? What makes using a line plot particularly useful? (See Sample Lesson 2, page 191.)

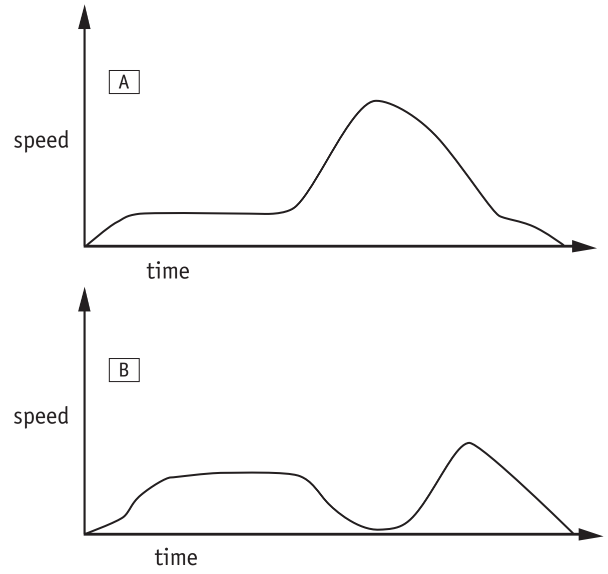
## *Did You Know?*

Students were asked to say which graph (see right) best matched the story. Of 64 Year 7 students, 76% said A, 13% said neither and only 11% gave B, the correct response. Typical reasons for choosing A or neither were:

- Because in graph B there are two hills and in the story there is only one hill, and it didn't say anything about a ditch in the road.
- Christie doesn't go down a hill after the long flat bit, she goes straight to a steep hill, but A has a little hill at the start and that's not in the story either.

Many students come to think of graphs as pictures and will look for a direct match between the shape of a plot and real-world physical features. As a consequence they have difficulty reading graphs that represent more abstract information.

Christie went for a ride on her bike. After she started riding, she went along a flat road at about the same speed for a while, and then she came to a hill. Her bike slowed as she climbed the hill, but when she got over the top, she sped up as she rode down the other side. She got so fast she had to put her brake on. Then she slowed right down until her bike stopped.




Graphs are visual representations of the *information we record* about objects, events or experiences (see Collect and Organise Data, Key Understanding 2). They are **not** pictures of the objects, events or experiences. Students need considerable experience in plotting data, including information such as distance, speed, strength and time, in order to understand what graphs show.

For example, students could take a walk that includes a steep hill and other events that influence their walking pace (such as a traffic crossing, or stopping at a shop). Using a pedometer they could record their distance from school at equal time intervals and make notes as they go. Plotting the actual data points can then act as a starter for discussing what graphs represent (see Interpret Data, Key Understanding 2).

KU 5

Part B: Summarise and Represent Data

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔

### Which Type of Graph?

Ask students to suggest which type of graph is more effective to get a quick overview of how the data is distributed. For example, they may choose to make a frequency graph or a frequency line plot to get an overview of the distribution of the measures of how far the balloon cars in their science experiment travel.

### The Average Kid

Have students measure their height to the nearest centimetre and record, pinning a label to themselves. Ask them to line up from shortest to tallest (by look) to consider what the 'typical height' is. (See 'The Average Kid', page 198.) They then rearrange themselves into a line plot with each column made up of students of the same height. Ask: What can we learn about our heights from this? Regroup into columns made up of various height ranges of, say, 2 cm, 5 cm and 10 cm and consider what each of the line plots do and do not tell us about our heights.


### Computer Graphing

Ask students to use a computer graphing program to explore different ways to group and graph measurements and compare the results. For example: Have them measure arm spans of students in Year 7 and Year 3, then produce four graphs grouping data in 1 cm, 5 cm, 10 cm and 20 cm intervals and compare. After they have experimented with groupings for several different sets of data, draw out that if the intervals are too small, it's difficult to see a pattern in the differences but if the intervals are too large it can look as if there are no differences.

### Pocket Money

Invite students to discuss when it makes sense to have the bars in a graph touching and when it doesn't. For example: The bars in a cumulative graph of pocket money saved month by month could touch, as could a height graph where heights are in intervals, but it wouldn't make sense for the bars in a graph of people's favourite food to touch. Have them brainstorm kinds of data and sort into those that could sensibly have touching bars and those that need to have separate bars.

### Sports Preferences

Ask students to explore which kinds of graphs make sense for a set of data, such as sports preferences of boys and girls. They enter their data in a computer graphing program and have the computer construct a bar graph, a pie graph, etc. Have students print and compare the graphs and discuss which are helpful and which do not make sense.

### Pie Graphs

Invite students to consider which type of data can sensibly be displayed in a pie graph. For example: They could collect the names of their classmates' favourite bands and singers and organise the information into the percentage of the class who preferred each band. When trying to construct a pie graph, they may realise that the percentages add to more than 100%. Draw out that a circle graph for this data can only make sense if each person in the class chose only one favourite band or singer. Whereas a bar graph makes sense even if more than one favourite can be chosen by each child, a pie graph must represent data that are about a complete whole (e.g. all the class) divided up into independent categories (i.e. each child is represented in only one category).

### Labelling Graphs

Ask students to consider when and what sorts of labels are appropriate when creating graphs. Ask: Would you need a title and label for the line plot you used to explore your data? Why? Would you need a title and labels for the graph you are going to put into the parents' newsletter with your report? Why?

## Later ✔✔

### Balloon Power

Have students consider which type of graph best suits their data about the distance their balloon-powered cars travelled. Have them use a range of sketch graphs or tallies to represent their data, and to describe the data from their graph. Ask: Are there any differences in what you can see about the data in the different graphs? Which graph gives you a better overview of the distance travelled by all the balloon-powered cars? Which one was the quickest and easiest to produce? Why? Draw out that a line plot is most suitable for discrete data that has a small range. Stem and leaf plots enable a large range of data to be quickly represented. (Link to Key Understanding 1.) A scatter plot could have been useful if students wanted to see if there is a relationship between distance travelled and some other measure, such as mass, size of balloon or diameter of the wheel.

### Publishing Reports

Provide opportunities for students to publish reports that include graphic representation of their investigations. Ask them to consider which graph to use and how much information to include.

# SAMPLE LESSON 2

**Sample Learning Activity:** Middle—'Choc-chip Cookies', page 186

**Key Understanding 5:** How we display our data depends on the kind of data we have and our purpose.

## Teaching Purpose

I wanted my Year 5 class to use line plots as a tool to allow them to observe and identify emerging patterns in data.

## Action

I held up a packet of chocolate-chip cookies and asked: *How many choc-chips would you expect to find in one of these cookies?* After drawing their attention to the range of their estimates, between five and 40, I issued a challenge: *Well, how are we going to find out?*

The students quickly decided that they could produce some data by eating the cookies and counting the numbers of choc-chips. As they had previously had some experience in representing data in different ways, I was pleased that they suggested they could organise their data in a tally or table and then make a bar graph.

However, I wanted to focus their attention on choosing a method for a specific purpose, so I asked: *Could we use a line plot instead?* William wasn't sure: *But isn't that just a graph anyway?*

*Well, line plots are graphs, but there are some important differences*, I said as I drew a line across the board, labelling five towards one end and 40 towards the other. *Think about how a line plot might be different*.

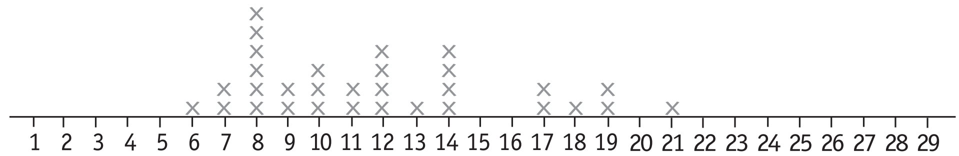
Maria: *You can do it faster.*

Abram: *It doesn't have to be as neat.*

*Yes, both those things*, I said. *So it's particularly good when we're organising our data. We can just build our plot as we go along—we don't have to make a tally or count how many in our categories first.*

The students each ate a cookie and took turns to enter a cross, representing their number of choc-chips, above the number line that I had drawn.

***The quickly produced line plots allowed the students to see the data accumulate 'before their eyes'. They were able to observe the changing 'shape' of the data as the sample size increased. They could not easily do this when constructing a bar graph.***




## Drawing Out the Mathematics

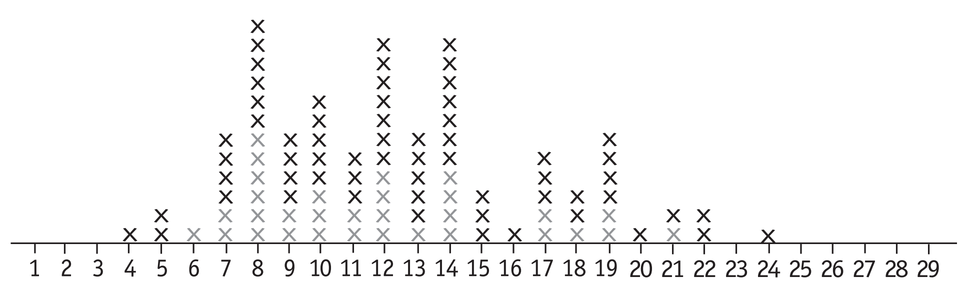
*So what does our data tell us about the number of choc-chips we could expect to find in a cookie?* I asked.

*The students had begun to consider the risks of drawing conclusions from a sample. (Link to Collect and Organise Data, Key Understanding 5.)*

Blair: *I reckon it's between six and 22*.
Te-Lin: *Eight had the most, so you'd mostly expect eight in a cookie*.

However, Julian was not convinced. *Yeah, but if we ate another cookie it might be different*. I asked Julian to elaborate and he said: *Well, it's just luck how many. If you ate another two and they had 14 then that would be more*.

I suggested we add some more data to our line plot and see what happens. They were more than happy to eat another cookie each, adding their new data to the line plot—this time using a different colour so we could distinguish between the two lots of data.




I wanted to draw my students' attention to the changing distribution of the data so I asked: *Is what we think now about the number of choc-chips the same as when we had only eaten our first cookie?*

*To further explore the idea of chance variation in a sample I decided to use line plots in a different way next lesson. The students could compare three separate line plots that each represented data produced from one cookie per student, rather than combine the data into one line plot.*

The students were able to make some comparisons.

Stella: *Well now it goes down to four and up to 24*.

Nigel: *And 14 is the same as 12. They're both the most*.

Denise: *And it's still like all the numbers between seven and 14 that have the most*.

I continued: *So, what do you think we would find if we added more data?*

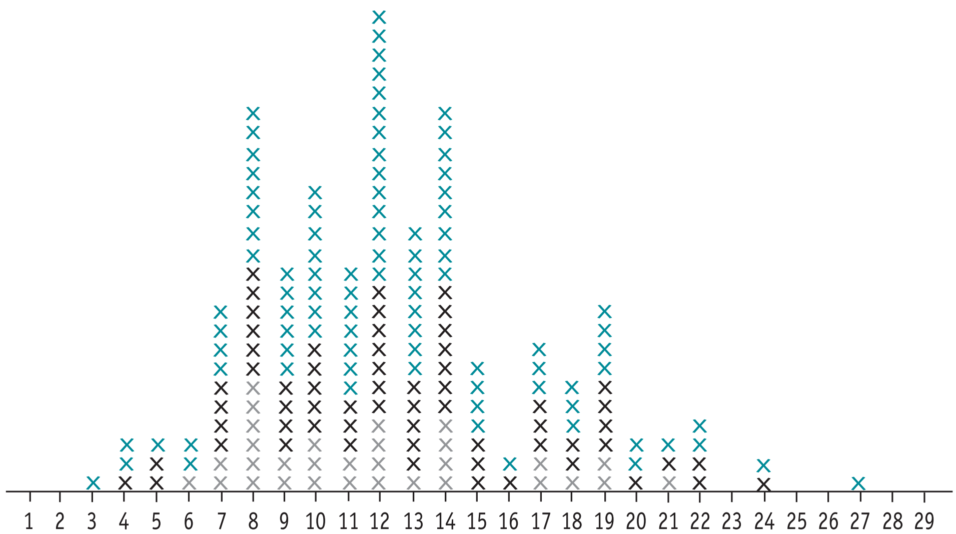
Peter answered: *I think it would sort of be the same, some of our crosses would be on the same numbers. But there might be some new numbers because the cookies might have different numbers of choc-chips*.

The students were eager to each eat another cookie and find out. They quickly recorded the additional data on the line plot using a third colour.

Gary: *Wow, look at that! It's changed. Twelve really is the most common number and there are more new numbers now.*

Denise: *But most is still seven to 14, it looks more bunched up.*

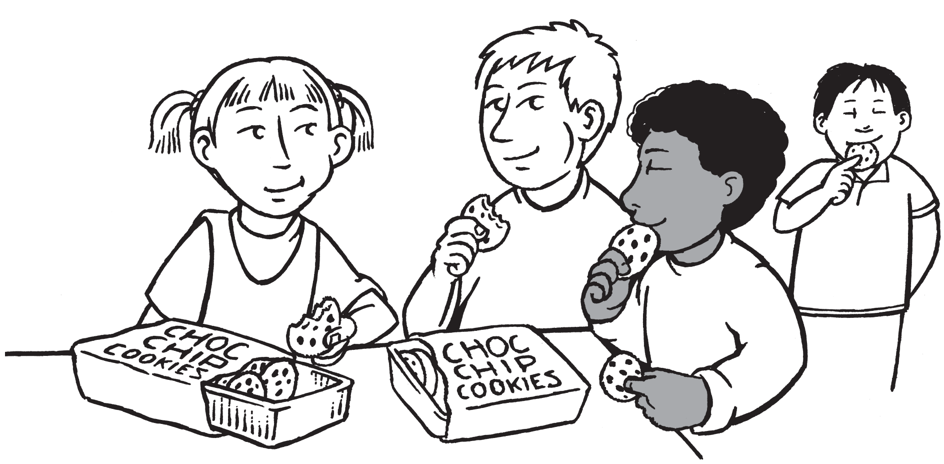
*Yes, I can see a pattern there*, said Peter.




*If a graph is to be interpreted without its producer being present, it must be sufficiently well labelled. However, fussing with details may not be necessary when we are using a graph for our own personal data analysis. Students need to take context and audience into account when making decisions about the type of data display they will use.*

## Reflection

After watching the line plot grow with each cookie, students were able to better understand why this graph was more useful than a bar graph for exploring emerging patterns in frequency data such as this. But in discussion they decided they would use a bar graph in their report about this activity, because it would be properly labelled and easier for someone else to understand.



## KEY UNDERSTANDING 6

### *We can use words and numbers to summarise features of a set of data.*

During the primary years students should learn basic approaches to summarising data. To summarise data means to describe, usually numerically, features of a whole group. Students need to learn that these 'summary' measures do not tell us about individual scores, but rather tell us something about the data set as a whole. Thus, when we summarise data we always lose some information.

At its simplest, summarising data may involve a simple statement that 'there are three blue birds and five yellow ones'. During the early and middle years, students will learn to make tallies within various categories and describe the frequencies obtained. Later, they should learn to summarise data with fractions and percentages (e.g. two-thirds of the class prefer dogs to cats; 30% of children and 50% of parents think school should go until 4 p.m.). This is often necessary when we wish to compare two or more different-sized groups.

An important feature of a set of data relates to the idea of 'being typical' or 'average'. Students need to understand that measures such as mean, median and mode do not tell us about individual scores or pieces of information, but describe characteristics of the group thought of as a whole. This is not easy to learn and simply working out these 'averages' using a formula is unlikely to help students understand their meaning. Exploring sets of data can help them develop a sense of what is 'typical' of it. Initially students will concentrate on individual pieces of data and find it hard to make sense of questions that ask them to consider measures of the group as a whole. They may pick the largest number when asked about what is 'typical' or 'average', or list the complete set of scores. Later they may invent measures that make sense, for example, suggesting that a typical score on a game was 'between 0 and 7' or 'it's mostly 3 or 4' or choose a score roughly in the middle of the data. Over time, they should come to understand the distinction between the three common measures of average: mode, median and mean—what each tells you and when it is useful. They should also informally investigate the effect on each of outliers (pieces of data that are very different from the rest) and zero scores.

Another important feature of a set of data is how spread out it is (the range of scores). Initially students may simply describe highest and lowest scores, which together with the median (the middle score) give a sense of the distribution of the data.

## Progressing Through Key Understanding 6

Initially students can summarise information by making simple counts. As students continue to progress they will efficiently count to describe and compare how many are in a number of different categories. Next they report numerically on the results of making tallies. As students progress further they use fractions, means, and lowest, highest and middle scores to summarise what their data shows. For example, they may say that about two-thirds of the children said they would like to play tennis if it were offered and that while the mean distance around heads was 55 cm, they varied from 49 cm to 60 cm. They could also suggest why average head size may not be helpful for designing hats! Later, students use fractions, percentages, means and medians to describe and compare their results. For example, having found that 22 out of 40 parents and 45 out of 60 students thought that teenagers should earn their pocket money, they convert each to a percentage to make comparison easier.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Describing the Group

Ask students to make statements that describe characteristics of groups of objects. For example: After a collection of shells has been sorted, ask students to describe the types and sizes of shells in the whole collection. Ask: What can you say about the collection of shells? Which shells in our collection have very different sizes? Which group are nearly all the same size?

### Breakfast Foods

After students have represented data about favourite breakfast foods, ask questions that require them to make statements about the data using, for example, a pictograph. Ask: How many children liked scrambled eggs? Encourage students to make their own summary statements without having to respond to your questions. Ask: What else can you tell from our graph/table?

### Mode

Build on students' intuitive notions of mode by posing questions such as: Which is the most common? Which do most people use? What is the most popular? How did you know?

### TV Programs

Scaffold summarising data into frequencies. Have students list the class names and record each person's preferred TV programs alongside. Ask: How can we rearrange this information so that we can see which program most students liked? What if we put the names of the programs on this side, and the students on the other side? It's taking us a long time to write all of these names. Do we want to know who likes each program, or just how many of us like it? Is there a quicker way we could record how many? Could we just count them?

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Dice Tossing

Have each group of students produce a large-scale line plot on the same topic, e.g. the results of tossing a die 30 times. Pin up the plots and ask students to pick one of the line plots and to think of how to summarise what the set of data looks like. Have students, one at a time, describe the data without saying which line plot they are describing. Other students try to decide which line plot is being described. Draw out informally that it helps to talk about where the data clusters, which scores appear the most, which are the smallest, biggest and about in the middle.

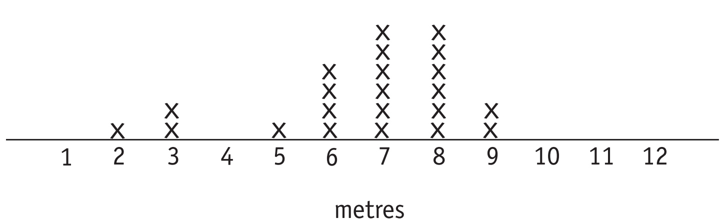
### Describing Data

After producing line plots of data, encourage students to invent ways to describe the general features of the set of data. They may say: *All the scores were between 2 and 10 and most were around 3 or 4*. Compare different students' descriptions, asking which they think are the best description.

### Estimating

Invite students to summarise line plots, trying to give a sense of the 'typical' result. For example: Given the plot below, ask students to describe the data to each other (see Interpret Data, Key Understandings 1 and 2). Ask: Did your description enable your partner to understand which were the most common estimates, or what was the range of estimates? Were there any estimates that you did not include in your summary because you knew they were 'way off'?

**Our estimate of how long the hall is**



## Middle ✔✔

### The Average Kid

Have students record their heights to the nearest centimetre. Ask: How tall is the 'average kid' in our class? After some discussion, have students line up around the room from tallest to shortest. Discuss where the 'average kid' might lie. Ask: Which student is in the middle of the line (the middle height in our class)? What is his/her height? Mention incidentally that this is often called the median, but it just means the middle result or score or height. Suggest grouping the height data by forming a human line plot. On the floor write the height ranges (e.g. in 3 cm ranges such as 130-133 cm) on an axis and ask students to form lines behind their range. Ask: Which line is longest? So more students in the class are between ... and .... Does it contain the middle score? Have students write an answer to the original question. Share descriptions. Draw out which descriptions focus on the middle score and which heights are the most common. (See Key Understanding 6, 'Later'.)

### Extend Average Kid

Encourage students to think of 'typical' in different ways. Using sticky notes rather than themselves, repeat a similar activity for other aspects of students' lives so that different children become average or typical for different things (e.g. family size, sport played, how they come to school). (See Key Understanding 5.)

### Arm Spans

Have students cut lengths of paper tape to match their arm span. Ask them to sort the tapes into piles of the same length (to the nearest centimetre). Ask: Which pile has the most strips? Which arm span is the most common? (This is the mode.) Is there more than one arm span that is most frequent?

### Identifying the Mode

Introduce the idea of mode as the most common result or score. Have students inspect several line plots and bar graphs and identify the mode. Provide them with unsorted data (of, say, 20 scores) and suggest that they organise the data in a way to make finding the mode easy.

### Stem and Leaf Plots

Have students look at data displayed in stem and leaf plots and determine the median and mode. Ask: How can we find the middle (median) amount? (*Count how many scores and then count halfway, or mark off pairs from each end of their data until we find the middle.*) How can we quickly see which is the mode or modes? (*The value that occurs most often.*) Use back-to-back stem and leaf plots to compare two sets of data and consider how the mode and median do, or do not, provide a useful way to compare the data.

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### School Uniforms

Have students consider proportion when summarising data from different-sized groups. For example: 16 (out of 20) Year 1 students and 16 (out of 32) Year 6 students voted for a particular design for a new school uniform. Discuss how to make a reasonable comparison. Ask: Is it more helpful to say 16 students in each class voted for the design, or that 80% of the Year 1 class but only 50% of the Year 6 class voted for it?

### Mean Averages and Fair Shares (1)

Suggest a scenario of buying jelly beans in bulk for a party and pouring a pile into each child's hand as they arrive. Is this fair? How can you redistribute so it is fair? Give each student in a group a handful of tokens (from five to 15). Ask them to make a note of how many they originally got and then find a way to redistribute their tokens fairly. How many did each get? (Groups will differ.) Each group reports to the class about how they did it. Most will physically put tokens together and redistribute, although some may count and divide. Ask: How sure are you it is now fair? If they all have the same (acknowledging some 'part tokens'), it is fair. How many tokens did your group have to start with? And how many students? How can you relate the total your group had to the number of students and the amount of each share? If the same total number of jelly beans were handed out to the same number of students, but distributed with different starting amounts, would the fair shares still be the same, or would they be different? Why? Ask students to test with their tokens if they are not convinced that the fair shares must be the same no matter how the total was distributed.

# Later ✔✔✔

### Mean Averages and Fair Shares (2)

Extend the previous activity to link the 'fair share' to the mean. Have students compare the share they got with their original amount. Draw out that some went up and some went down. The share was 'sort of' in the middle of their original amounts. Have students write the number of tokens they got on a sticky note and produce a line plot. Again draw out that the fair share was 'sort of' in the middle of the original amounts. Tell students that the 'fair share' they have worked out is another 'typical score' or average and it is called the mean.

### Mean Averages and Fair Shares (3)

Extend the previous activities to emphasise the underlying meaning of the 'mean'. Informally discuss the difference in amounts students started with and ended with. Draw out that some went up, some went down, but the fair share removed the differences between students. Ask students to make a table listing their names and showing the number of tokens they started with and the number they finished with. Each calculates the difference, either using + and – , or the words 'more' and 'less' to signify the direction.

| Name | Start Amount | Fair Share | Difference |
|---|---|---|---|
| Terri | 8 | 9 | + 1 |
| Tony | 11 | 9 | – 2 |

Have students combine the differences, taking into account whether they need to add or subtract. *(The result should be 0 if the mean was exact, or if there were a few 'left over' the total will reflect that.)* Again emphasise that the mean balances out the differences—it is another way of thinking about the middle or typical amount. Help students find the median (middle score) and the mode for the data and compare to the mean. What information has been lost in these summaries? How might this kind of information be useful? *(Comparing large quantities of overlapping data to see if we can say there is an overall difference between groups)*

### Mean Heights

Put students into groups of four or five and challenge them to find a way to get their average (mean) height without calculating. Offer paper tape to help. *(They might lie on the floor in a line head to toe, mark the beginning and end, stretch the tape along it and fold into four or five equal parts OR they might use the paper tape to find each height, stick them together and fold, etc.)* Have students in each group line up in order of height and put their mean height in its right place. Discuss where it sits and again draw out the idea of balancing the differences.

### Line Plots (1)

Provide each student in a group with a different line plot, each containing 20 to 25 dots spread over similar but not identical range. Select line plots so that, say, two have the same mode but different other averages, two have the same mean, etc. Ask each student to write a description of his or her data set, keep the description and then put the plot into a pile in the middle of the table. Each student in turn reads their description of the data set to the group, and the group decides which plot it describes.

### Line Plots (2)

Vary the previous activity by having each student work out the mode, median and mean for their data set and write it on a separate sheet of paper. The groups make two piles, one of the plots and one of the list of three averages for each, and then together sort them so that the line plot is with the correct set of averages. Ask students to discuss how they could tell. Pin the plots and summaries around the room and have a class discussion to draw out features of the plots. Ask: For which plots were the three averages about the same (look at the shape or distribution of the data)? What is the effect of single scores that are 'odd' (look at outliers)? How would it help to know the range (i.e. the highest and lowest values)?

### Comparing Data (1)

Ask students to summarise data to compare the same information from different populations. For example: Have students collect data (such as height, time spent watching TV) from the whole school and represent the data in a way that allows comparison. Ask: Is the data from your class like the whole school? Where does most of the data lie for each group? How is it the same or different? Where is there no data? Is this the same for each of them? Is there some data out on its own? Is this the case for each of the sets of data? How would finding the average or typical quantity (i.e. mode, median and mean) help with this?

### Comparing Data (2)

Extend the previous activity and use census and other population data found on the Internet to compare their class data to data from schools both nationally and internationally.

## Later ✔✔✔

### Balloon Power

Have students use the highest, lowest and middle scores or measures to describe data. Ask them to record the distance travelled by each student's balloon-powered car on a word-processed table or spreadsheet on the computer. Sort the distance column from shortest to longest distance. Find the middle measure (median). If there is an odd number of measures, one will be in the middle. If there is an even number of measures, the median is halfway between the two 'middles'.

### Families (1)

Have students produce data sets that result in a specific value for the mean. For example: Ask them to draw a series of families so that the mean number of children in the families is three. Compare drawings. Ask: Is there more than one way to draw these families? Draw out that different data sets can result in the same mean.

### Families (2)

Extend the previous activity by asking students to include a family with no children. Ask: If this family didn't have any children, what changes would we have to make to the rest of the families?

### Families (3)

Extend the previous activities further by using a mean of, say, 2.5. Ask: Is it possible for any of the families to have 2.5 children? If not, how can we make sure the mean number of children is 2.5? Draw out that the mean doesn't need to represent the value of any one particular member of the data set.

### Sports Equipment

Ask students to consider how the median helps describe only some data. For example: Have students try to find the middle measure for sports equipment or other sets of data with categories that do not have a set numerical order. Ask: Why is it possible that one student says balls is the middle when someone else says it is bats? Is the median helpful when the categories of sports equipment can be in different orders? Draw out that because the categories could be listed in any order, a middle measure doesn't make sense. Students indicate whether a median is a helpful descriptor as they deal with other data.

### Predictions

Use a class frequency graph of the students' predictions for future events to compare mode and median. First, ask whether we can see which was the most popular prediction. For example: How many babies will be in the guinea pig's litter? How many goals will the winning team get? Is there more than one popular prediction? Then have students find the middle (median) prediction. Ask: Which best described the 'average' prediction?

### More Choc-chips

Extend 'Choc-chip Cookies' (page 186) by recording the quantity of choc-chips in each cookie on a sticky label. Use the data to explore and compare the mean, mode and median (see Sample Lesson 3, page 204).

### The Average Kid

Repeat 'The Average Kid' (page 198), recording each height on a sticky note. Put them in order and find the middle height (median). Reorganise the notes to form a line plot and identify the height or heights with the most students (the mode or modes). These measures provide a sense of the typical or average measure for the whole group. Ask: Is the mode the same as the median? Close? Then have students round their heights to the nearest 5 cm, make a new sticky label and line plot, finding the median and mode/s as before. Ask: Has the median changed? Has the mode/s changed? Why?

# SAMPLE LESSON 3

**Sample Learning Activity:** Later—'More Choc-chips', page 203

**Key Understanding 6:** We can use words and numbers to summarise features of a set of data.

*Soon after the students began to produce their data they started to pose questions such as: Do I count this half choc-chip as a 1 in my tally? I used the opportunity to talk about the need for consistency while collecting data. (See Collect and Organise Data, Key Understanding 4.)*

## Teaching Purpose

My Year 7 students were able to organise data in frequency graphs and identify the 'most common' category. To develop their sense of what was 'typical' of a set of data, I wanted to focus their attention on the meanings of and the differences between mean, mode and median.

## Connection and Challenge

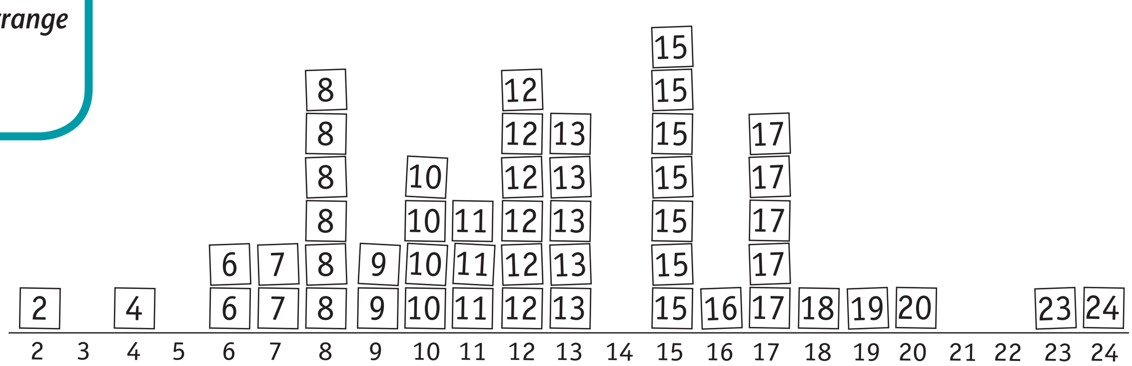
During language lessons the class had begun to question the validity of some claims made by manufacturing companies. They showed interest when I suggested we could use mathematics to help evaluate such claims. Holding up a packet of chocolate-chip cookies, I asked them to imagine they were the manufacturers of the product and to decide what advertising claims they could make about the number of choc-chips in their cookies.

## Action

*I suggested two cookies per student to ensure that there was sufficient data to allow trends to emerge.*

The students decided that they would need to find the number of choc-chips in some cookies. So, at my suggestion, they were happy to each carefully nibble two cookies and record the number of choc-chips in each cookie on separate sticky notes. When I asked if there was a way we could organise our data, the students suggested a line plot.

*I deliberately chose to use sticky notes to collect and organise our data so we could physically rearrange the data later in the lesson.*



After constructing our line plot, I asked: *What advertising claim could we make about our choc-chip cookies if we used this data?*

*Most of our cookies have 15 choc-chips,* said Penny.

*No they don't!* argued Danny. *Seven out of 50 isn't most. Fifteen's just the most common number. We could only say that our cookies are more likely to have 15 choc-chips than any other number.*

I continued: *We can use the most common number as an average or typical number to give people an idea about the whole set of data without telling them all the individual numbers.*

*But I thought the average was 13!* Jerome argued. *Thirteen is in the middle of two and 24.*

*Well, we could have a look to see if 13 is the number in the middle of our data,* I said.

*I recognised that I would need to return to these ideas of chance in future lessons.*

*Some students have a misconception that the 'average' is the middle number of the range of data.*

*Students often do not realise it is possible to find the mode, median and mean of the same set of data.*

## Drawing Out the Mathematics

I invited some students to rearrange our set of sticky notes into one long line, in numerical order, across the board.

2 4 6 6 7 7 8 8 8 8 8 . . . 12 12 12 12 12 13 13 13 13 . . . 17 17 17 17 18 19 20 23 24

When they were finished I asked, *How could we find the middle score?* We decided we could remove the sticky notes in pairs, one from each end, until we got to the middle. *What would that actually be doing? Is there an easier way?* I asked. We established that halving the number of notes and counting from one end would do it.

*Twelve is the number in the middle!* said Jenna.

*Yes,* I agreed. *What does the 12 tell us about the number of choc-chips in our cookies?*

I was pleased when Dallas ventured: *It's like the halfway number. Half the cookies have 12 or less choc-chips and half the cookies have 12 or more choc-chips.*

Gabby then said: *Well, that wouldn't make the cookies sound very good in an advertisement!*

Inja replied: *Yes, but it tells us what they are like.*

I agreed. *That's right, we can also use the middle number as an average or typical number to tell people about the set of data.*

Jerome was satisfied. *I get it now. I didn't know you had to get the middle of all the numbers. I thought it was just the middle of two and 24, like on a ruler.*

However another student had a query. *I thought the average was when you added up and divided.*

*We actually ended up with two 'middle numbers' because we had an even number of sticky notes. In this case both 'middle numbers' were 12. If the two middle numbers had been different, I would have discussed the notion of an imaginary 'middle number' halfway between the two.*

*Many students (and often the media) use the word 'average' to refer to the 'mean'.*

*When the class calculated the mean to be 12.25 some students were perplexed:* **But that's silly! You wouldn't count out exactly 12.25 choc-chips for each cookie.**
*I explained that the mean is a calculated, imaginary number that we use to give us some information about the group of cookies as a whole rather than about individual cookies. Many students were convinced that the mean had to be one of the numbers of choc-chips.*

*At this stage I wanted to help the students to understand that there are three common measures of the average—mode, median and mean—and to understand each of them. It was not my intention to provide them with formulas for their calculation.*

I replied: *You might be thinking about another average or typical number of a group called the mean. Imagine our 50 cookies before they were cooked. How could we find the number of choc-chips we would have had if we had removed all the choc-chips from the cookies?*

Pointing to the sticky notes, Sarah answered: *You'd have to add up all those numbers.*

I continued: *Then, imagine we decided to put the choc-chips back into the cookies. But this time we want every cookie to have exactly the same number of choc-chips so each cookie got its fair share. How could we find the number of choc-chips for each cookie?*

Harry offered an answer: *We'd have to get that number we found when we added and then share it into 50 bits.*

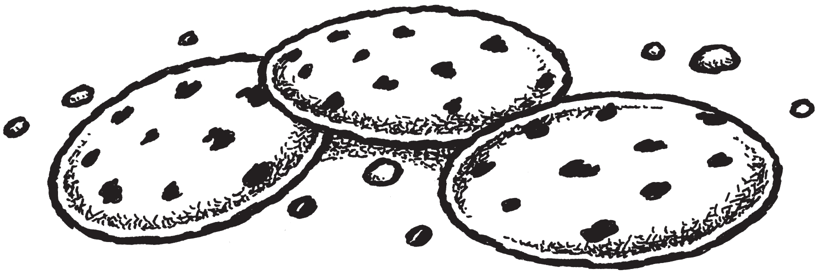
## Reflection

*So would we use the most common number, the middle number or the mean in an advertising claim?* I asked.

*Well, the mean doesn't tell people very much,* Irene suggested. *It's not what's really in the cookies. I think the most common number is better because that's what people are most likely to get.*

*But the median tells the truth more,* argued Hillary.

I was pleased. The students had begun to compare the information provided by the mean, mode and median. I now needed to provide opportunities for them to consider which 'average' was the most useful in describing other sets of data to answer particular questions.



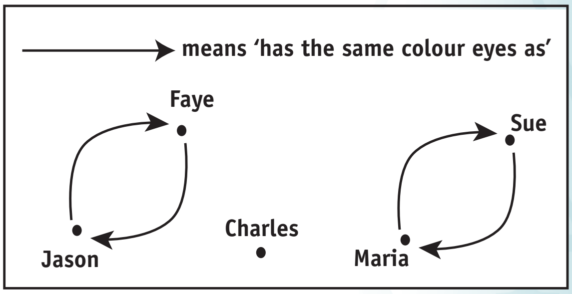
# BACKGROUND NOTES

## *Types of Data Displays*

### Arrow Diagrams

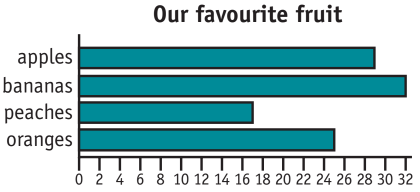
(See Key Understanding 4)

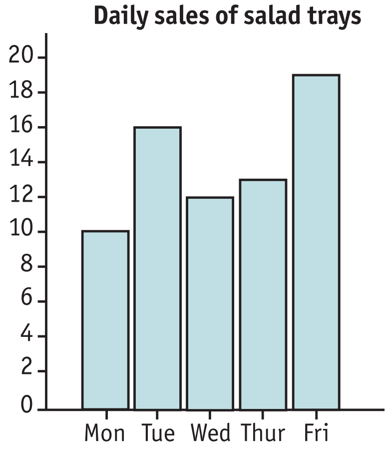
Used to organise ideas, categories and relationships. Symbols for objects, people and so on are spread out on a page and arrows are used to link the symbols in meaningful ways. The display needs to have a legend or key that shows how the arrows should be interpreted.

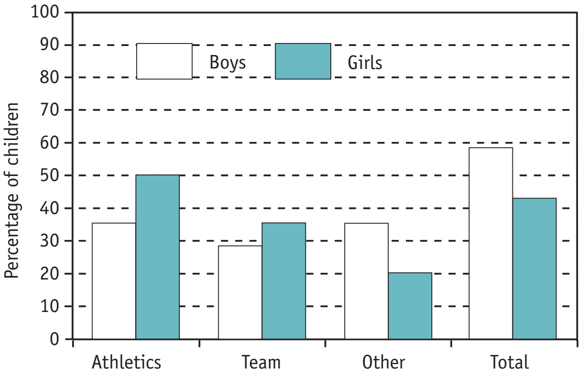



### Bar Graphs

(See Key Understanding 1)



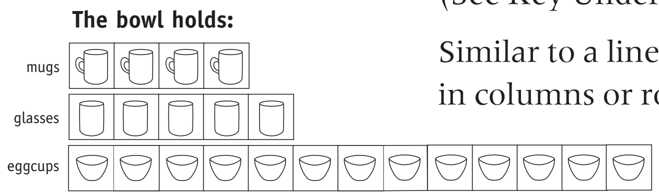




A common graph type that uses the lengths of columns or rows to represent frequencies or measurements of categories or groups.
A wide range of data can be represented, with either axis being used for categories or groups, and the other axis calibrated as a scale to show a count, a percentage or a measurement. The lengths of the bars should be proportional to each other and where the data is about discrete categories, the columns or rows (bars) must be separated. Different sources of the same kind of data can be compared by putting two or more bars side by side and providing a key to show the meaning of adjacent bars. Also see Histograms.

## Block Graphs

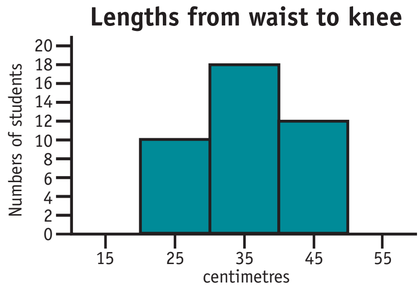
(See Key Understanding 1)




Similar to a line plot in that each piece of data is displayed in columns or rows of squares above or beside a baseline. Because each square counts as one, a second axis is not required, similarly to a Line Plot.

## Histograms

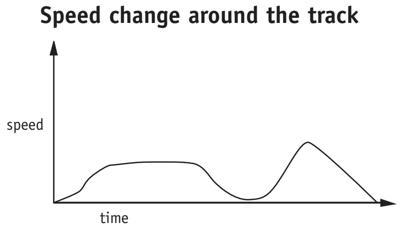
(See Key Understanding 1)

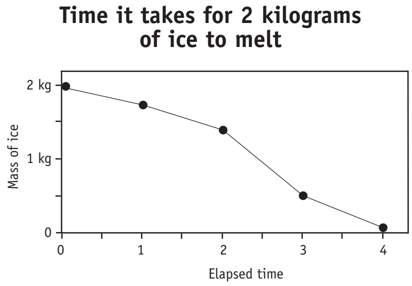



A variation of a bar graph used for continuous quantities, or categories that can be thought of as naturally ordered in time or quantity. Bars are generally vertical, with the columns touching to represent the continuity between the groups of data. If different intervals of data are used, the bars may be different widths, so that the area of the bars is proportional, as well as the length.

## Line Graphs

(See Key Understanding 2)

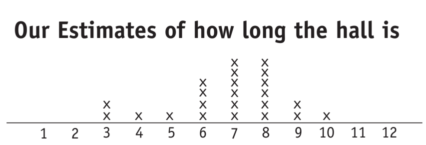



Used when it is meaningful to think of the frequency or measurement varying, usually over time. Points are plotted at intervals and the points joined to represent how the quantity changes between the data points. The base axis must be calibrated as a measurement scale so that every point on the line has meaning.

## Line Plots (also called Dot Plots)

(See page 156)


Used to record or display frequency data. Dots, crosses or other equal-sized marks are used to represent each piece of data. They are placed above a baseline that has been labelled for each category or number. A second axis is not needed because each 'dot' represents one piece of data.

## One-way Tables

(See Key Understanding 4)

| Name | Eye colour |
|---|---|
| Mai | blue |
| Freya | brown |
| Peter | blue |
| Kim | brown |

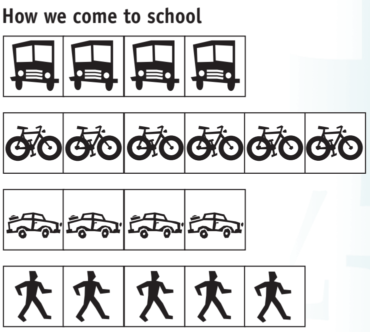
| Eye colour | Frequency |
|---|---|
| blue | //// |
| brown | // |
| green | / |

Sets out related information in adjacent lists. Only the columns in one-way tables are labelled. The list of names or information in the first column links in rows to other information across the page.

## Pictographs

(See Key Understanding 1)

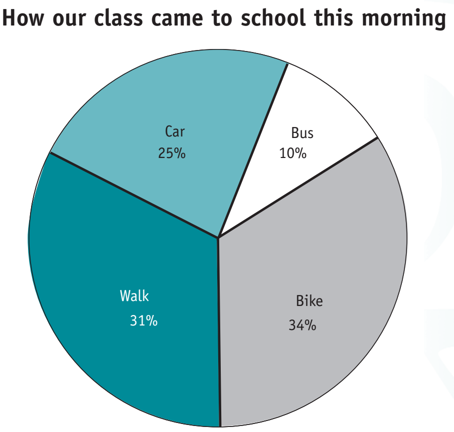
Small pictures or icons that relate to different categories of data are placed equidistant from each other in rows or columns. Each icon may represent one or more pieces of data. A key will indicate if an icon means a quantity of data, in which case in between quantities are represented by parts of icons.




## Pie Graphs

(See Key Understanding 1)

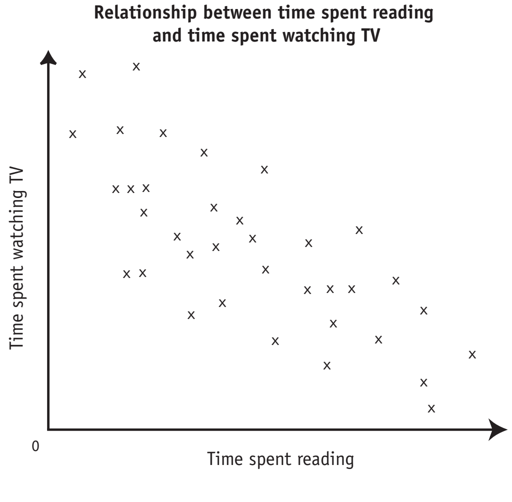
Circular graph in which the area of the slices relate proportionally to the quantities in the categories. Its use only makes sense when each piece of data belongs in exactly one category of a clearly defined whole set of data. The quantities are normally shown as percentages.

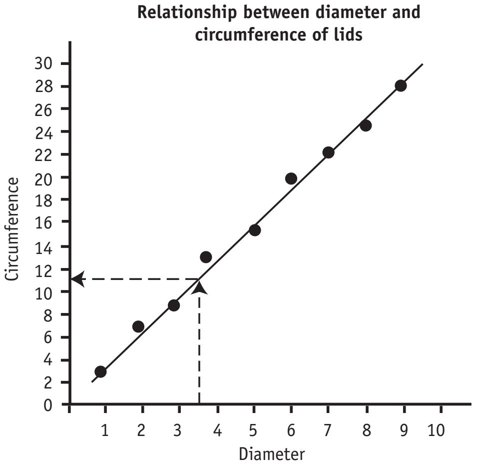



## Scatter Plots

(See Key Understanding 2)

Demonstrates visually how two different types of data are related. Two different measures for the same person or thing are paired and plotted on a two-axis grid. Each axis relates to one type of measure, and a mark shows where the two measures intersect. A 'line of best fit' is sometimes used to emphasise strong relationships between the data.



## Stem Plots (also called Stem and Leaf Plots)

(See Key Understanding 1)

Used to display numerical data that range from zero into the hundreds. The tens are listed in a column, with the units of each piece of data listed in order in rows to the right and/or to the left of the respective tens number. It enables the full data set to be visible, while creating a graph of the data grouped in tens. This form of display makes it easy to see the mode/s and to work out the median.

| | |
|---|---|
| 0 | 3 4 |
| 1 | 0 3 5 |
| 2 | 1 1 3 4 7 |
| 3 | 0 2 2 3 4 5 7 9 |
| 4 | 0 5 6 |
| 5 | 1 3 3 3 4 9 |
| 6 | 3 |
| 7 | 0 |

| Girls | | Boys |
|---|---|---|
| 2 1 | 0 | 3 4 |
| 6 | 1 | 0 3 5 |
| 7 4 4 0 | 2 | 1 1 3 4 7 |
| 8 8 4 2 2 1 1 | 3 | 0 2 2 3 4 5 7 9 |
| 3 0 | 4 | 0 5 6 |
| 9 4 4 4 3 1 | 5 | 1 3 3 3 4 9 |
| 7 6 1 1 1 | 6 | 3 |
| | 7 | 0 |

## Two-way Tables

(See Key Understanding 4)

Information is shown in two or more categories in columns and rows, which may or may not need to be totalled. It is useful for showing how different types of frequency data might be related, for example, the different ways that boys and girls travel to school.

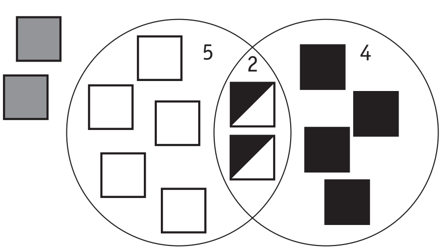
| | Heads | Tails |
|---|---|---|
| Heads | HH | HT |
| Tails | TH | TT |

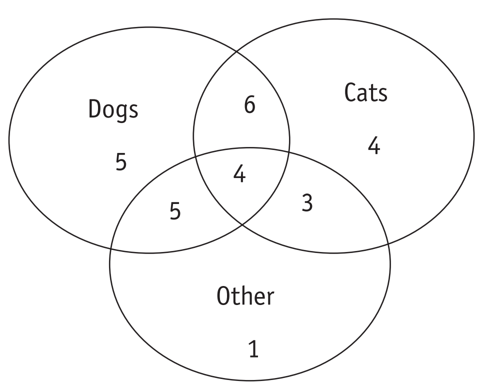
| | Girls | Boys | |
|---|---|---|---|
| Year 6 | 15 | 13 | 28 |
| Year 7 | 12 | 18 | 30 |
| | 27 | 31 | 58 |

## Venn Diagrams

(See Key Understanding 4)

Used to visually represent data that have overlapping categories. The number in each category is shown within an oval, and the number that is in more than one category is shown in the relevant overlapping sections.



CHAPTER 6

# Interpret Data

This chapter will support teachers in developing teaching and learning programs that relate to this outcome:

***Locate, interpret, analyse and draw conclusions from data, taking into account data collection techniques and chance processes involved.***

## *Overall Description*

Students interpret and report on their own data (about age and height, pet preferences, pocket money, fitness levels, attitudes to smoking, a weather simulation) and data taken from a variety of secondary sources (magazines and newspapers, sports results, farm records). They locate and use databases about Australia and Australians, such as those available through the Australian Bureau of Statistics. They distinguish between census and sample data and understand that considerable care needs to be taken in selecting samples and in forming conclusions about the whole group from sample data. In this, they realise that the uncertainty involved in drawing conclusions from data is the essence of the connection between 'chance' and 'data'.

Students know that when making use of data they should question their quality and credibility, and the way in which they are organised and represented before evaluating the conclusions drawn by others. They recognise that good-quality data and a knowledge of chance process can help them assess risks, form opinions and make decisions such as, for example, whether or not to immunise children. However, they also know that while mathematics and data can each contribute to our decision-making, they cannot determine what we should or should not do in any particular circumstance, the latter being matters of personal and/or community judgment.

| Markers of Progress | Pointers<br>Progress will be evident when students: |
| --- | --- |
| Students describe what their own and classmates' displays of data show. | • read and compare frequencies from lists and simple one-way tables, e.g. *This shows that four people like apples and six like bananas*<br>• interpret block graphs produced by others, e.g. *say Their graph shows the results of their data collection, e.g. Their graph shows that most children they asked like tacos better than burritos*<br>• describe how their graph shows the results of their data collection, e.g. *We spun the spinner ten times. These three squares on our graph show the three times we got yellow, these (pointing) show we got red six times and green once. Blue didn't come up*<br>• write a few sentences to describe the results of their data collection, e.g. *In our 20 tosses we got more heads than tails; We found that pizza was the most-liked food*<br>• comment on information in displays of data produced by themselves and peers, e.g. *By looking at all the graphs we can see that some groups got more tails but others got more heads* |
| Students read and make sensible statements about the information provided in tallies and in simple tables, diagrams, pictographs and bar graphs. | • report the frequency information provided in a tally produced by a classmate, e.g. *Their group got seven 6s and only three 2s*<br>• interpret straightforward one- and two-way tables, e.g. *This shows that of the 17 berries, ten were green and edible, seven were green and not edible*<br>• read frequencies from a bar graph (with each unit on the frequency axis marked) and hence describe the data, e.g. *Their graph shows that 11 of the people asked said they preferred frozen yoghurt*<br>• interpret pictographs produced by others where each picture represents more than one unit, e.g. *This shows that there were a few more than 40 dogs and almost 30 cats*<br>• explain their own data displays to their peers, talking about the features represented, e.g. *My tally shows that the highest score was 10 and the lowest score was 6*<br>• comment on their predictions in light of the results of their own data collection, e.g. *We thought that other children would find the same thing scary but that isn't what we found—we now think that what makes you scared might change as you get older; We thought that blue Smarties would come out least and we were right* |
| Students read and make sensible statements about the information provided in tables, diagrams, line and bar graphs, fractions and means, and comment on how well the data answers their questions. | • describe information from diagrams that may include arrow diagrams, tree diagrams, Venn diagrams or Carroll diagrams, e.g. *This shows that six people like pizza but not hamburgers, eight like hamburgers but not pizza, two don't like either and 14 like both*<br>• read the information provided on axes of bar and line graphs, including where all calibrations on the scale may not be labelled<br>• interpret and report on information provided in tables and bar graphs where data are grouped into simple intervals that can be regarded as categories, e.g. waist measurements may be grouped into intervals to reflect tracksuit pants sizes<br>• interpret and report on information provided in line graphs, informally describing trends in the data, e.g. *This shows that we raised more money each week until the last two weeks when …*<br>• comment sensibly on how well their own collected data answers their original questions, e.g. *We thought asking people what food they liked would help plan the camp, but we didn't ask it very well and so we couldn't classify it. Next time we would …*<br>• comment sensibly on how well their questions are answered by data provided by, or collected from, others, and how the data might be improved, e.g. *We wanted to know whether people preferred books that had pictures in them, but the data doesn't classify books that way so it wasn't very helpful. We needed to know …* |
| Studentsread and make sensible statements about trends and patterns in the data in tables, diagrams, plots, graphs and summary statistics, and comment on their data collection processes and their results. | • interpret bar graphs/histograms for grouped data, including where the scales on the axes must be 'read' between calibrations<br>• informally interpret relationships and reach conclusions from scatter plots, e.g. *It looks like the people who read fastest usually read the most, but we can't say that one causes the other*<br>• check accuracy of their data before interpreting it, e.g. *We plotted the surface areas of the cubes against the volume of the cubes, and found that we could draw a smooth curve through all the points except one, so we checked and found we'd made a mistake*<br>• distinguish between different averages in their interpretations of data, e.g. *These data show that the mean time taken by the class to run around the oval has improved over the term but the median has not. This probably means that half the class didn't get better but the people who started off already in the top half got better*<br>• write or present an accurate summary of the information displayed in a range of tables and graphs, e.g. *what is shown by their line plot of ages of the caregivers of class members* |

## Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

| Key Understanding | Stage of Primary Schooling—Major Emphasis | KU Description | Sample Learning Activities |
|---|---|---|---|
| **KU 1** Graphs, tables and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them. | Beginning ✔✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 214 | Beginning, page 216<br>Middle, page 218<br>Later, page 221 |
| **KU 2** When we analyse and interpret data we are deciding what it says and what it means. There is a difference between the data itself and what we think it means. | Beginning ✔<br>Middle ✔✔✔<br>Later ✔✔✔ | page 228 | Beginning, page 230<br>Middle, page 232<br>Later, page 234 |
| **KU 3** We need to evaluate the data we are using in order to be confident about the conclusions we have drawn. | Beginning ✔<br>Middle ✔✔<br>Later ✔✔✔ | page 240 | Beginning, page 242<br>Middle, page 244<br>Later, page 246 |

**Key**

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

## KEY UNDERSTANDING 1

***Graphs, tables and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them.***

There are three related aspects to this Key Understanding:

- to make sense of (that is, to read) data displays, we need to know what aspect of the real-world situation the data refers to (link to Collect and Organise Data, Key Understanding 2)
- data displays are not pictures, they do not 'look like' the real-world situation from which the data was produced (link to Summarise and Represent Data, 'Did You Know?', page 187)
- we need to learn how to read graphs, tables and diagrams.

Beginning in the primary years, students should learn to read a range of diagrams, tables and graphs that go beyond those they can readily produce for themselves. The range should include those that provide very good models of data presentation, through those that are flawed in some way, to those that are downright misleading, reflecting the variety one would find in general use. Older students should locate and use data from a wide variety of media sources.

Visual and tabular displays of data require specific reading techniques that students must learn. For example, tables are generally based on a cell or grid structure and conventional left-to-right, top-to-bottom scanning may be unhelpful. In reading a table such as this:

| Input | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Output | 3 | 6 | 9 | 12 | 15 |

students need to understand that the 'Input' and 'Output' numbers are intended to be thought of as pairs (relating back to the situation from which the data comes). Therefore, the table needs to be read in an order that connects the pairs, perhaps noting that the output number is always three times the input number. However, certain aspects of the relationship will also be highlighted by reading along the rows, perhaps noting that the input goes up by ones while the output goes up by three. Often the meaning of a table is lost because students do not read them either flexibly or as intended. Depending on how they are read, however, the relationships could 'pop out' or not be noticed at all.

Students need to understand the conventions about how the content of a table cell is determined and labelled. They should practise reading one-way and two-way tables, including where both cell frequencies and row and column totals must be read and where some grouping of data is involved. They could be asked, for example, to extract particular pieces of information from tables (How many girls play a particular sport?) or to explain what a particular number in a cell is about (What does that 11.6 tell us?).

Many students are unable to separate their reading of a graph from their personal knowledge of the situation. Thus, students may ignore the 'obvious' information, such as that red is the favourite colour, in favour of statements about their own preferences. To overcome this, students should read graphs about situations for which they are not already privy to the key information.

Graphs also have their own conventions and students need structured activities that require them to describe or recount the *information* provided in a graph. Even reading bar graphs requires that students read labels properly and read frequencies and measures from a range of scales, including reading between calibrations. Students seem to respond to some graphs intuitively and can read them without specific instruction, e.g. where bar graphs represent the heights of children. Perhaps the most difficult for students to read are those that involve two variables and when the graph's appearance doesn't directly match the idea it represents (see Summarise and Represent Data, 'Did You Know?', page 187). Students should learn to write brief stories to describe what is represented by such graphs. The aim is for students to understand that such graphs are intended to help us get a feeling for how variables are related to each other, they are not pictures of situations.

## Progressing Through Key Understanding 1

Initially students can extract frequencies from lists and one-way tables. As students continue to progress they can read a tally, extract data from simple one- and two-way tables and work out frequencies from a pictograph or a bar graph where each unit is marked on the axis. Next, students read frequency and other types of information from a range of tables and bar and line graphs (including where data has been grouped). As students progress further they are able to extract information from a wide variety of descriptions, tables, diagrams and graphs produced by others, including informal graphs showing the relationships between two quantities.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔✔

### Shoelaces

Encourage students to read lists used to display information in the classroom as a natural part of other activities. For example: Display a list of the names of students who can tie shoelaces and encourage students to find someone on this list when their shoes need to be tied. (See Collect and Organise Data, Key Understandings 2 and 3.)

### Colour Graph

Invite students to read block graphs produced collectively by the class. For example: Have students create a graph of favourite colours in their class by placing coloured blocks in lines. Ask: How can we tell which colour most people like? Can we tell by just looking? What if we wanted to know how many people liked red? How can we tell? (See Summarise and Represent Data, Key Understanding 1; link to ***First Steps in Mathematics: Number***, Understand Whole and Decimal Numbers, Key Understandings 1 and 2.)

### Reading Block Graphs (1)

Ask students to read block graphs produced by their classmates containing information to which they are privy. For example: Have students produce a block graph that represents information on a topic familiar to the class. They exchange graphs with a partner, who describes what the graph says. (Link to Summarise and Represent Data, Key Understanding 1.)

### Reading Block Graphs (2)

Ask students to read block graphs produced by their classmates containing information to which they are not privy. For example: Each group works on a different question for their graph. They then hand their graph over to another group, which reads the graph to decide what it is about. Each students states one piece of information from the graph.

### Reading Pictographs

Invite students to read pictographs in which one picture stands for one object, e.g. where faces are used to represent the number of students in the class who belong to a particular house or faction. Using a label for each house, students place their pictures next to where they belong. Ask: How many students in gold house? Can we tell from our graph which house has the most people?

## One-way Tables

Have students read frequency information from one-way tables produced by their classmates. For example: Ask students to collect information about the number of each colour jelly beans in a bag and record the information in a table. They swap with a classmate who 'reads' the information. Ask: How many red jelly beans were in your partner's bag? Which colour did she or he have most of? Pin up a table of data, point at a number and ask: What does this number tell us?

## Lunch (1)

Ask students to produce a simple table that lists the names of students in their group (of six or eight) and indicates whether they brought lunch from home or ordered lunch. Have one student in the group say a name, and another read to find what that child did. Draw out that it can be quite slow to find the right name. Ask: How could we make it quicker? Draw out that listing alphabetically might help. (Link to Key Understanding 3; Summarise and Represent Data, Key Understanding 4.)

## Lunch (2)

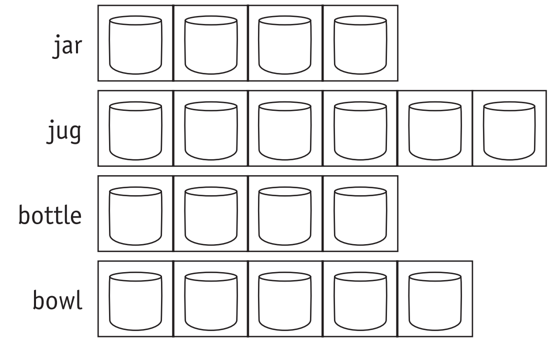
Extend the previous activity by having students swap reworked lists. Call out names and have students find whether the name is on their list, and read the relevant information. Ask: What if it were a whole class list? Draw out that alphabetical order would help.

## Tallies

Have students read tallies to say how many in each category. For example: While on an excursion, ask students to use a tally to record each time they see a particular animal. Afterwards, ask: How many cows did you see? Encourage children to skip count by 5s. Extend this to include tallies that are not their own.

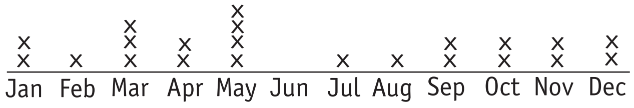
## Measurements

Have students record measurements of different objects using the same unit in a table or in a picture or block graph, and help them to interpret it. For example, use a cup to measure how much water various containers hold. Ask: Which container holds the most water? Which holds the least? How much more does the jug hold than the bottle?




## Birthdays

Invite students to read line plots to compare how many children have birthdays in different months. Ask: How many birthdays in April?



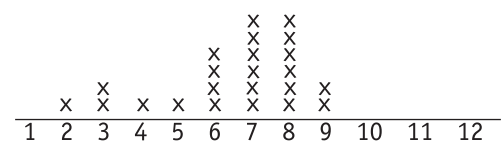
# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔✔

### Estimating

Give students practice in reading line plots. For example: Given the plot shown at the right, ask them to say what it is about. Ask: How many children think the hall is 5 metres long? … 8 metres long? … 11 metres long? … 4 metres long? What is the longest estimate? … the shortest? (See Key Understanding 2; link to Summarise and Represent, Key Understanding 6.)

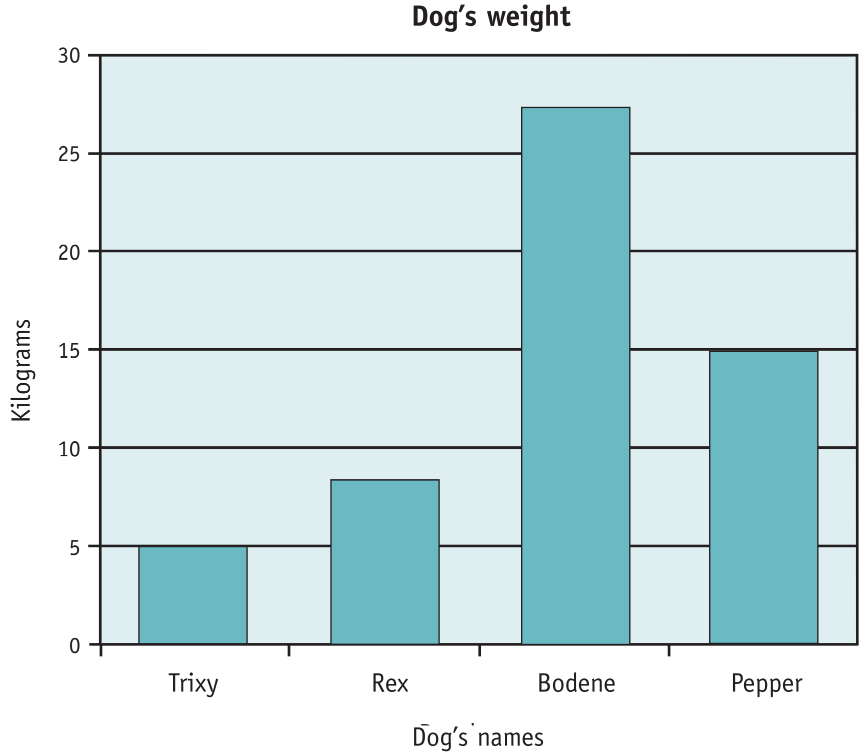
**Our estimates of how long the hall is**




### Dogs

Have students read simple bar graphs in which scaffolding, in the form of horizontal lines, assists in the reading. For example: Model the process of reading the graph below, by asking students to tell their partner what the heading and each axis tells you. Ask them to say, without referring to the numerical scale, which dog weighs the most. Ask: How can you tell just by looking? Students then look at the numerical scale to say how much each dog weighs. Provide new graphs and ask students to describe to their partner what they say.

**Dog's weight**



### No Props

Ask students to read simple bar graphs in which the reading 'props' are not provided. Repeat the previous activity, but with bar graphs in which horizontal lines are not drawn in.

### Pie Graphs

Invite students to interpret simple pie graphs by comparing the sectors and saying which is the largest/smallest.

### Counting Cars

Have students skip count to read pictographs where one picture stands for more than one object. For example: Students count by 10s to work out how many. Ask: If one car stands for 10, how do we count half a car? How can we tell which has the most? Do we need to count?

### Tennis

Ask students to read simple one-way tables from published sources to find specific information. For example: Use two separate tables, one showing heights and the other weights of tennis players. Ask: What does this table tell you about? *(How much each player weighs.)* How much does (player X) weigh? Who weighs the most? Are different players the same weight? Are there three players who are the same weight and also the same height? How can we tell? *(Look at other table.)*

### Shoe Size

Invite students to extract the information they need from a frequency table produced by others to produce a block or bar graph. For example:

| Survey of children's shoes sizes | |
|---|---|
| **Size** | **Number of Students** |
| 4 | 4 |
| $4\frac{1}{2}$ | 6 |
| 5 | 0 |
| $5\frac{1}{2}$ | 7 |
| 6 | 3 |

### Getting to School

Have students read simple two-way tables created by others, and answer questions that require them to interpret the contents of various cells.
For example:

| | Year 4s | Year 7s |
|---|---|---|
| Walk to school | 6 | 12 |
| Drive to school | 18 | 8 |
| Ride to school | 4 | 10 |

Ask: What does the 18 tell us? How many in Year 4 walk to school? Do more students walk to school in Year 4 or in Year 7? How do you know? How do most students get to school in Year 7? (See Key Understanding 2.)

## Middle ✔✔✔

### Venn Diagrams (1)

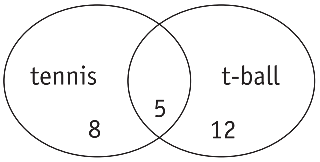
Ask students to read Venn diagrams involving two circles. For example: Have them place their names in a Venn diagram according to whether their family has boys or girls. Ask: Which circle are you in? Who else is in the same circle as you? What about the people in the middle? What does their position tell you about their families?

### Venn Diagrams (2)

Extend the previous activity by replacing names with frequencies and have students explain what each number says. Draw out that the number of families having boys is the number having only boys, added to the number having boys and girls. Repeat for girls.

### Venn Diagrams (3)

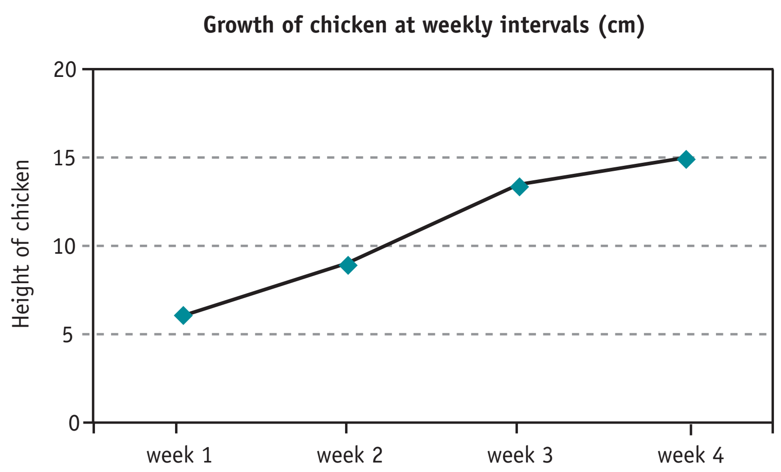
Provide two circle Venn diagrams for students to read frequency information from, e.g. the number of students who play each of two sports, tennis and t-ball.




Point to the frequency for tennis alone and ask: What does this number tell you? How many people play tennis altogether? How many play t-ball altogether? Point to the 5 and ask: What does this number tell you?

### Chickens

Have students read simple line graphs to say how a quantity varies over time, e.g. the growth of a chicken at weekly intervals. Ask: What do the labels on each axis show? How tall was the chicken after week 2? … after week 3? So, how much did it grow between weeks 2 and 3?

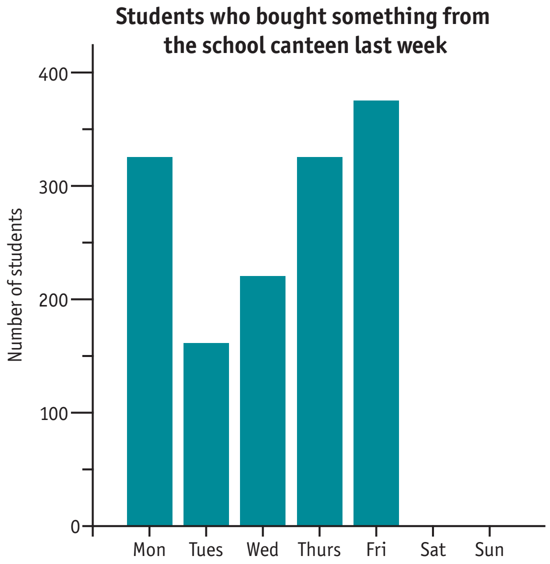


# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

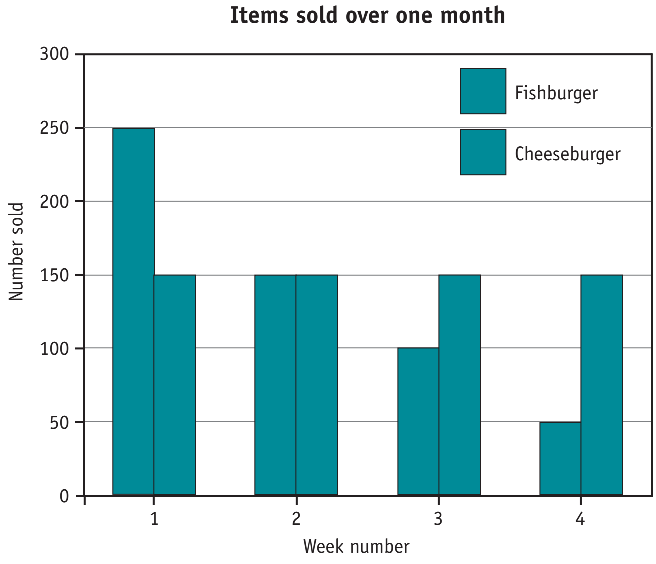
### Estimating Values

Ask students to read simple bar graphs where all the calibrations on the frequency axis may not be marked. Help them to estimate values between calibrations.




### Double Bar Graphs

Have students read horizontal and vertical 'double' bar graphs. For example:




Ask them to describe to their partner what the heading tells them. They then read the information on each axis and explain what each shows. Ask: What do the two different coloured bars tell you? Which burger was most popular in the first week? Did it continue to be popular? What do you notice about the sales of cheeseburgers over the four-week period?

### No Props

Repeat the previous activities with bar graphs that do not have 'props' in the form of horizontal bars.

### Statements/Collecting Graphs (1)

Ask students to collect bar graphs from newspapers, magazines, encyclopedias and the Internet. Ask them to select a graph and make two factual statements about what it shows. Ask their partner to agree or disagree about whether the graph actually says that.

## Later ✔✔✔

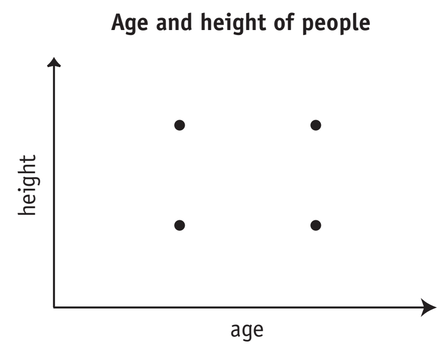
### Collecting Graphs (2)

Extend the previous activity and ask students to select a graph, write two questions for their partner to answer from the graph.

### Collecting Graphs (3)

Ask students to work in groups to sort their collected graphs into those that were easy and those that were hard to read. Ask: What made the difference?

### People Graph




Provide students with two-variable graphs with just a few points to 'read'. For example: Present a picture of four people (short and old, short and young, tall and old, tall and young) and a matching graph (see right). Explain that the four points represent the four people and challenge the students in groups to write the name of each person on the graph. Ask students to justify their placement to the whole class. Draw out that the graph shows the height and age of the four people. Ask: What does the vertical axis show? … the horizontal axis? What happens to the heights as you go up the vertical scale? What happens to the ages as you go to the right? Describe each person's age and height. (See Summarise and Represent Data, Key Understanding 3.)

### Dot Plots

Provide students with a range of two-variable dot plots, each with just a few dots on it and ask them to state what each point shows, for example, how tired and happy four students were after the sports carnival. Have students match the points with alternative forms of the information, e.g. a drawing such as in the previous activity, or a table showing the data.

### Tennis Table

Ask students to read one-way tables from published sources, including where there are several columns of information. For example: Use a table showing height, weight and age statistics for tennis players. Ask: What does this table tell you about? *(Each player's weight, height and age)* How much does (player X) weigh? Who weighs the most? Are there different players the same weight? Are the three players who are the same weight also the same height? How can we tell?

## Tennis Graph

Invite students to extract data from one-way tables such as in the previous activity, to produce a two-variable dot graph. For example: Students find the height and weight of each player and plot points in a two-way coordinate grid to represent the extracted information. Ask them to compare what the table and graph highlight. Repeat for weight and age, and height and age. (See Summarise and Represent, Key Understanding 3.)

## Pocket Money

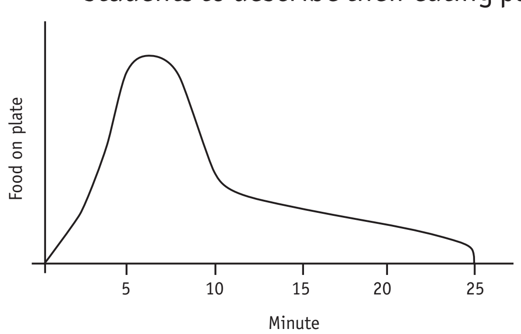
Have students read two-way tables where the data is grouped into categories.

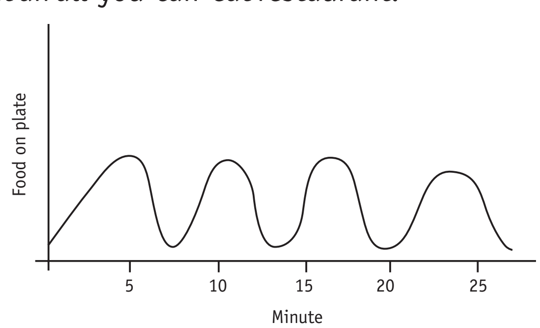
| Pocket money earned by students in Years 5 and 6 | | |
|---|---|---|
| **Amount ($)** | **Number in Year 5** | **Number in Year 6** |
| 0 to 5 | 8 | 10 |
| 6 to 10 | 12 | 1 |
| 10 to 15 | 3 | 6 |
| 16 to 20 | 0 | 1 |

Ask: What does the 10 mean? What does the 12 mean? How do the table headings help you to know? What other questions can you ask of the data?

## Eating Patterns

Ask students to read and report on graphs where the appearance doesn't directly match the idea it represents. For example: These graphs were drawn by students to describe their eating pattern at an all-you-can-eat restaurant.




Ask: What does the graph show about each child's eating pattern? How did you use the information on both axes to help you tell the story?

## Junk Food

Have students bring in published graphs. Choose a graph that has not been fully labelled. Enlarge it and, as a class, try to work out its meaning. Ask: What might the columns be about? How does looking at the main title help? What else do you need to know to interpret the graph? (See Sample Lesson 1, page 225.)

## Lunch

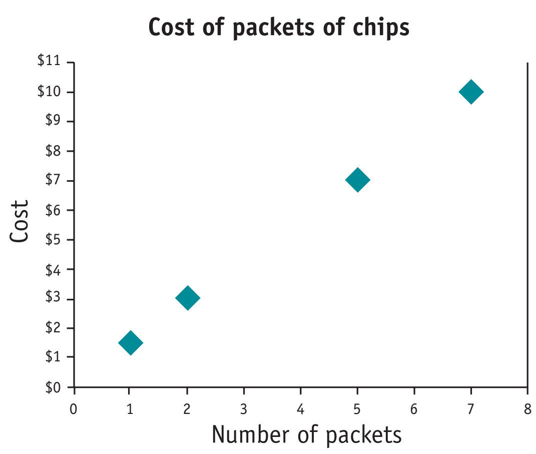
Have students read two-way tables where rows and columns are totalled. For example:

| Lunch orders for the carnival | | | | |
|---|---|---|---|---|
| | Year 5 | Year 6 | Year7 | |
| Pies | 5 | 7 | 4 | 16 |
| Pasties | 7 | 3 | 5 | 15 |
| Sausage rolls | 10 | 10 | 15 | 35 |
| Lasagne | 4 | 5 | 7 | 16 |
| | 26 | 25 | 31 | |

Ask: How many of each item of food needs to be ordered for the sports carnival? Write the number of items that need to go into the basket for each year level so we can check that the number of items is correct.

## Buying Chips

Invite students to read two-variable (coordinate) dot graphs. Ask: What does this graph tell you about? How much does it cost to buy five packets? How many packets can you buy for $2? Is there a discount for buying seven packets? How do you know?

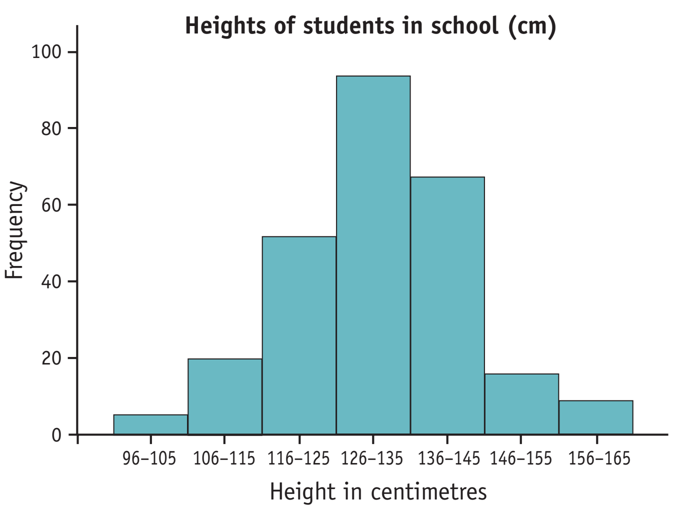



## Venn Diagrams

Have students extract information from Venn diagrams as in the 'Middle' Sample Learning Activities, but involving three circles.

## Heights of Children

Ask students to read graphs where the data is grouped and where reading between the calibrations on the frequency axis is required. Ask: How many students are between 126 and 135 cm high? How did you work it out?



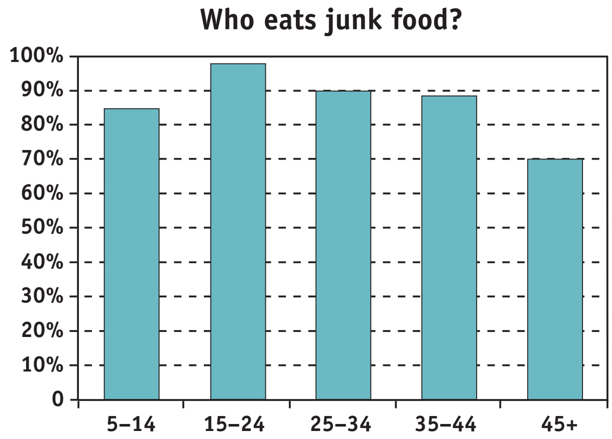
# SAMPLE LESSON 1

**Sample Learning Activity:** Later—'Junk Food', page 223

**Key Understanding 1:** Graphs, tables and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them.

## Teaching Purpose

During a health lesson on nutrition in my Year 5 class, Amanda brought in this graph that she had found.




She wanted to include it in her report about junk food, but she was not sure what some aspects meant.

## Action

I enlarged the graph on the photocopier, placed it on my easel, and invited students to comment on it. Amanda said, *I don't know what the numbers mean—they didn't put anything on the side or bottom to tell you.*

Amanda didn't bring in the accompanying article, so I asked the students what the columns might be about.

*It could be how much they eat,* ventured Katrina.
*Or it might be how much it costs,* offered Serena.

I suggested they look at the main title for a clue. *It says 'Who eats junk food?' so it's more likely to be about groups of people.*

*Students can make comparisons between columns without understanding what exactly the data represents. They often try to relate the size of the columns to real-world quantities—in this case, the quantity of junk food.*

*I realised at this point that Ariel's idea that it was about how many people were in the survey was not surprising at all. For most of the surveys the students had undertaken, their bar graphs showed exactly that—they asked a question, categorised the answers and graphed the number in each category. This was possibly the first time they had needed to think about the bars representing a proportion or part of a whole.*

*Understanding percentages is not expected until Level 5, but I had noticed that many graphs and tables in the texts and reference books they were currently using did in fact involve percentages. Helping students to think of 85% as '85 out of every 100' (even if initially they ignore the 'every') will enable them to get some meaning from these displays. This will not conflict with their later proportional understanding of 85% as 85/100. (See **First Steps in Mathematics: Number**, Understand Fractional Numbers, Key Understanding 7.)*

Amanda then correctly reasoned that the numbers across the bottom might be ages. I clarified this for those who did not understand. *The bar labelled 15 to 24 is about the group of people who are at least 15 years old but not more than 24 years old.*

*It's how much junk food the people eat,* Rod claimed confidently.
*Yes, the 15 to 24 group eat more junk food than kids,* said Amanda, *and old people eat the smallest amount.*

Although the students were comparing the relative heights of the columns, their language suggested they were thinking about the bars as simply representing amounts of junk food. I wanted to help them think more broadly about the kind of data the bars could represent.

## Drawing Out the Mathematical Idea

*But is the graph about how much junk food they eat, or is it about who eats junk food?* I asked.

*If it's about people then the numbers could tell us how many people were in each of the groups,* suggested Ariel. *Like there are 80-something kids, nearly 100 in the next one, 90 in the middle ones, and 70 old people—if you added it up you'd get how many people were there altogether.*

I realised that even if they understood the vertical axis was about people rather than food, a lack of understanding of percentages made it difficult to make sense of what the numbers on the vertical axis meant. However, I thought they might be able to understand that the height of the columns showed how many people out of every hundred in each age group ate junk food. *You can think of the 85% as 'out of every hundred children from 5 to 14, 85 of them eat junk food.'*

*So there's a hundred people in each group, and the columns show how many eat junk food—so does it mean the rest don't eat junk food?* asked Amanda.

*Yes, out of EVERY hundred people in each group, that's how many eat junk food, so what's left from 100 is how many out of EVERY hundred don't eat junk food.*

Few saw the full implications of 'every hundred' in my explanation but thinking of percentages as 'out of a hundred' would be sufficient for them to compare the categories.

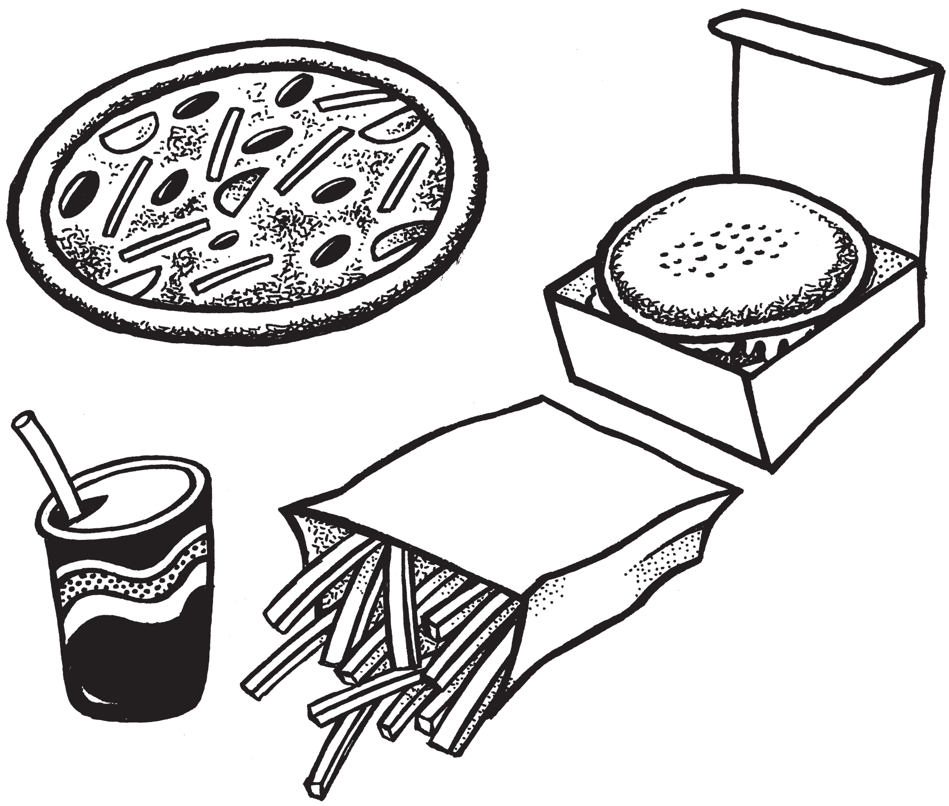
## Reflection

To focus their attention back on the possible conclusions they might draw from the data display (see Key Understanding 2), I asked: *Do all people eat junk food?*

*No,* said Evan, *most people in all the age groups eat junk food, but some don't.* Others agreed and I was satisfied most students had developed some awareness of the percentage scale.

However, I realised we had not talked about what aspect of the situation the data might refer to, so I asked: *How do you think it was decided if a person ate junk food or not?*

This stimulated discussion about the information we couldn't find out from the graph alone, e.g. which foods were considered junk food, what sort of survey was carried out, which questions were asked. Amanda said there was a 'story' with the graph, so I asked her to bring it in the next day to see if it would help us make better sense of the data.



KEY UNDERSTANDING 2

***When we analyse and interpret data we are deciding what it says and what it means. There is a difference between the data itself and what we think it means.***

Analysing and interpreting data goes beyond direct reading (see Key Understanding 1) and involves deciding what the data tells us and what we think it means. This includes interpreting the data produced and interpreting beyond the data produced.

## Interpreting the Data Produced

This is involved when we interpret the data that is actually available. Students may ask their classmates which pets they like best and, on finding out that eight children say 'rabbits', four say 'dogs' and fewer say 'other pets', conclude that rabbits are the most popular pet. This involves them in two processes of interpretation:

- Firstly, rather than simply reporting the information (eight children say 'rabbits', four say 'dogs', …), they need to compare the frequencies for each category in order to say which has more. In other circumstances they may also need to combine and integrate information, perhaps calculating totals or differences or reorganising data to enable a question to be answered.
- Secondly, either implicitly or explicitly, they have to infer 'popularity' from 'pet liked best'. Had they used a different measure of popularity (e.g. asking which pet children would get if given a choice), they may have reached a different conclusion. Often we do not distinguish clearly between the data itself and our interpretation of it, and this can lead to miscommunication and inappropriate actions.

## Interpreting Beyond the Data Produced

This is involved when we draw inferences and make predictions that go beyond the data we have.

- Firstly, we generalise to a group bigger than the sample for which we have data. Students should begin to make inferences on the basis of samples—informally considering whether it is reasonable to generalise. To do this, they need to tap into their existing knowledge for information about whether this sample is likely to be a good predictor of the population. For example, they may decide to use the pet preferences of their own class and to generalise to local children of similar age, but not to much older or younger children or to children from different backgrounds.

- Secondly, we may generalise a sameness, difference, pattern or trend in our data, drawing an inference about the nature of the relationship between the variables, and hence about what will happen in other cases or in the future. We might look at a scatter plot, for example, and search for trends—to see if there is a positive relationship between the two measures, or a negative relationship or no relationship at all.

Through the primary years, students should begin to take chance variation into account in sensible rather than technical ways. They should learn not to assume that what happens in a sample will exactly predict what happens in the whole population, and to develop an everyday sense of what is normal variation and what is unusual. For example, they might find that 50% of boys and 47% of girls in their class prefer watching TV over other pursuits. This is just one more boy and probably does not imply a meaningful difference between boys and girls. The uncertainty involved in sampling is what causes the uncertainty in drawing conclusions. This is the reason we should be more conservative in interpretations based on data collected on samples, than if the whole population of interest has been surveyed or tested. This is what links 'chance' and 'data'.

## Progressing Through Key Understanding 2

Initially students reach simple conclusions based on counting, saying, for example, *There are more strawberries so strawberries are the best-liked fruit.* As students continue to progress they can describe what their own data collection shows and can comment on the reports of their peers. Next, students interpret tables, diagrams, bar graphs and pictographs produced by themselves and others, including their peers, drawing sensible conclusions from them. They also comment on their predictions in light of their collected data. As students progress further they interpret fractions and means, informally commenting on trends they notice in their own and others' data. Later, students describe the results of their data collection, talking informally about relationships they see in the data. For example, they may note that faster readers also seem to read the most, but state that they do not know which causes which.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Fruit

After students have lined up their fruit to form a simple block graph of fruit types, model questions to help them to interpret their display. Ask: Are there more bananas or more apples? Which fruit is there most of? Which fruit is there least of? Which two lines of fruit are the same length? What does that tell you? Tell your friend one other thing our display of fruit shows. How many more are there in this row of bananas? So how many more people brought bananas? Select a fruit not in the display (say, pears) and ask: How many people brought pears? How can you tell? Repeat regularly with other similar displays of collected objects. (Link to Summarise and Represent Data, Key Understanding 1.)

### Fruit Graphs

After students have produced block or picture graphs that represent, for example, their fruit or other collected items, repeat the previous activities, modelling the processes of interpretation of simple graphs (as distinct from physical displays of objects). Say: There were more bananas than apples. What do you think this means? Does it mean bananas are more popular than apples? What else might it mean?

### Interpreting Displays

After students have carried out activities to interpret their own visual displays, have them swap graphs and interpret each other's.

### Birthdays

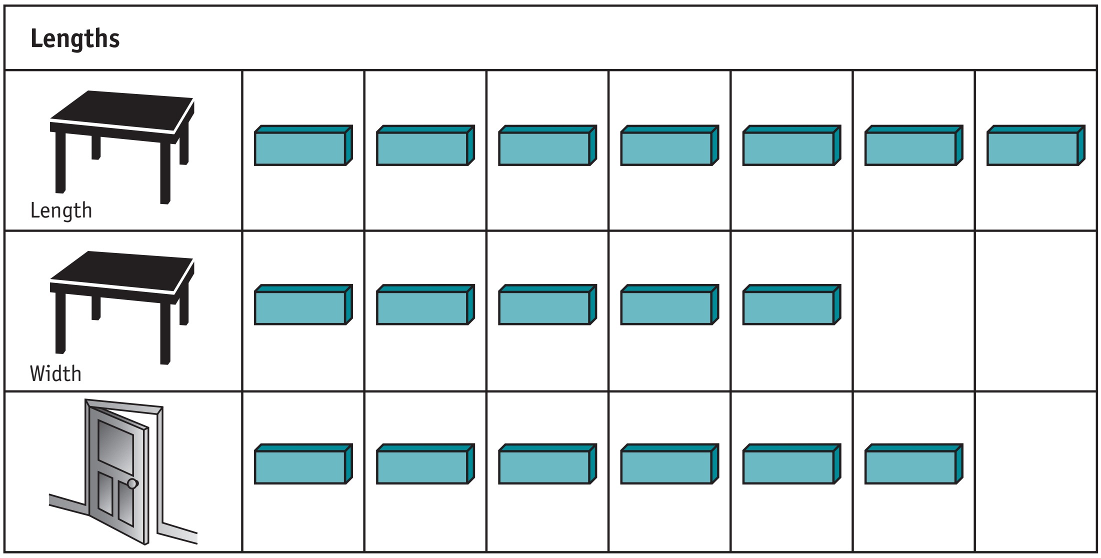
After students have produced a class block graph representing the month of their birthday (see 'Birthdays', page 217), model simple questions as for 'Fruit' (above). Extend by helping them interpret ages from the graph. Ask: Does being in the same month mean you are the same age? Which students are likely to be the oldest? Which students have already turned 7? How many have not had their birthday yet?

### Measurements

Extend 'Measurements' (see Key Understanding 1) by asking: If you were really thirsty, which container would you drink from? If they were full of medicine that tasted awful, which container would you choose? (see Summarise and Represent Data, Key Understanding 1.)

## Does It Fit?

Have students record measurement data as graphs, then help them to draw simple conclusions. For example, ask them to measure a table and a doorway using blocks.




Ask: How many blocks fit across the table? How many fit along the table? So is the table wider or longer? How many blocks fit across the doorway? Is the doorway wider or is the table wider? So would the table fit through the door? (Link to ***First Steps in Mathematics: Measurement***, Understand Units, Key Understanding 3.)

## My Family

Have students produce a picture graph showing the number of people in their family, then help them to interpret it. Ask: How many families have four children? Which are more common, families with two children or with four children? Which is the most common family size? Do any of the columns have the same number of pictures in them? What does that mean? How many families have no children in them? Does that mean there are no families with no children? What do you mean by ‘family’?

## Predictions

After students have read frequency information from one-way tables or simple bar graphs, ask them to make predictions. For example: Have students collect information about the number of each colour jelly bean in a bag. Ask: Does this mean that all bags of jelly beans will have more red ones?

# SAMPLE LEARNING ACTIVITIES

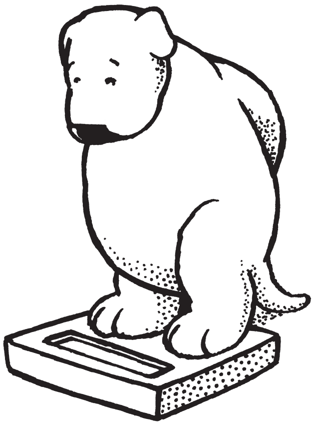
## Middle ✔✔✔

### Estimating

Have students interpret line plots produced in 'Estimating', page 214. Ask: What are the most common estimates? Does this graph give you any ideas about what the real length of the hall might be? What is the difference between the longest and the shortest estimates? Would you expect people to disagree by that much? What does that suggest to you? *(Perhaps people have different ideas of how long a metre is.)* (See Key Understanding 1.)

### Dogs

Provide students with simple bar graphs and model the process of interpreting the data. For example: Have students read the basic information from the graph of dogs' weights (see 'Dogs', page 218). Ask: Which two dogs are closest in size? How much more does Bodene weigh than Trixy? Which dog is likely to eat the most? Why do you think that? Do you know it from the graph, or are you deciding this for yourself? Can you be sure?



### Zero Frequencies

Provide students with bar graphs in which at least one category has a zero frequency. For example, in interpreting a graph of sport preferences, ask: Were any sports not preferred by anyone? How does the graph show that? Why did they not just leave that sport off the graph? Refer to a sport not on the graph and ask: How many people prefer that? Can you tell? If it isn't on the graph, can you interpret that?

### Venn Diagrams

Extend 'Venn Diagrams (3)' (page 220) by asking students to compare the frequencies in the different categories. Ask: So do more people play tennis or t-ball? Do more people play one sport or do more play two sports? How many people play at least one of these two sports?

### Getting to School

Extend 'Getting to School' (page 219) by having students combine the information in the simple two-way tables to answer questions. Ask: Do more students walk to school, drive to school or ride to school? Are there more students in Year 4 or Year 7? How do most of these Year 7 students get to school? What is the least used method of getting to school?

### Playground Equipment

Have students gather data by collecting or counting things. For example: Ask students to count the number of people on different pieces of equipment at recess. Ask: Which equipment was used the most? Which equipment do people like the best? Draw out the difference between the two types of answers. One is factual information we can be certain about, the other is a conclusion we can say is probably true but we are not sure about. Ask: Are there other reasons people may have played on the equipment? Do you sometimes play on equipment that isn't your favourite?

### Chickens (1)

Invite students to compare measurements in simple line graphs that show how a quantity varies over time, e.g. the growth of a chicken at weekly intervals. Ask: Did the chicken grow more between weeks 1 and 2 or between weeks 3 and 4? How much has the chicken grown altogether? (See Key Understanding 1.)

### Chickens (2)

Extend the previous activity by comparing two graphs showing the growth of different chickens. Ask: Which chicken grew the most? Which grew the fastest? What is the difference in the height of the two chickens?

### Dog Food

Invite students to draw inferences from graphed data to answer a question, e.g. 'Which food do dogs like best?' Ask them to look at their graphs showing which foods dogs mostly eat. Ask: Does this mean that most dogs prefer to eat chicken? What else might influence what a dog eats? Have students write to a dog food company describing their results.

### Canteen Orders

Have students make statements of likelihood based on data in a simple bar graph that shows more children order sandwiches from the canteen than pies. Ask: Is it likely that children prefer to eat sandwiches, or is there another reason why more children order them? What might the graph look like if the canteen's bread order didn't arrive? What might the graph look like on a cold day?

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Pictographs

Ask students to examine a pictograph, e.g. of vehicles passing the school, where each picture represents more than one unit. Ask questions to help students interpret the data:

- What type of vehicle passed the school most/least often?
- How many more trucks than cars passed the school? What did you have to do to work it out?

Ask questions that require students think beyond the data:

- At what times would you expect more bikes to pass the school?
- Would you expect the same results every day of the week or every hour of the day?

Draw out the difference between the data and the conclusions we draw from the data.

### Lunch

Extend 'Lunch' (page 224) to interpret the data. Ask:

- How many of each item of food need to be ordered for the carnival?
- How many lunches need to go into each year's lunch basket?

Have students say which numbers they used and what they did to the numbers to work it out. Ask questions that require students to interpret beyond the data:

- Would you be able to use this data to say what to order for the swimming carnival in summer?
- How would you change the menu for the next winter carnival?

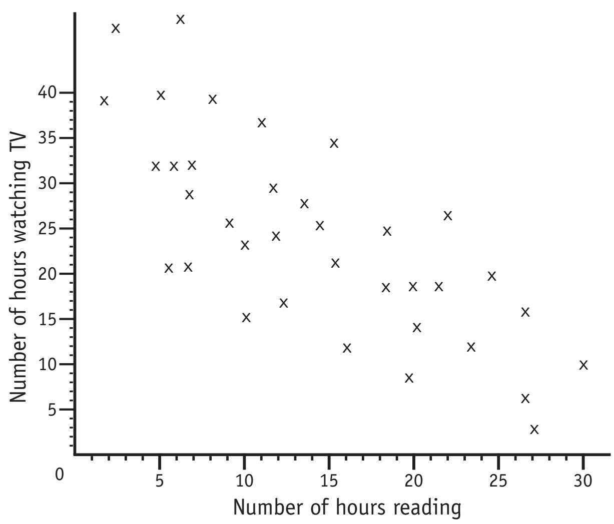
### Heights of Children

Have students interpret graphs where the data is grouped and where reading between the calibrations on the frequency axis is required (see 'Heights of Children', page 224). Ask: What height are most children in the school? How many are taller than 145 cm? How many are between 146 and 165 cm? How did you work it out? Can you see a trend in the height of the children in the school? Who might this information be useful for? *(People ordering school uniforms or chairs and desks)*

## Ordered Pairs

Encourage students to look for trends and make predictions from data. For example: Ask them to make a graph, labelling one axis 'TV time' and the other axis 'Reading time' and plot an ordered pair (TV time, reading time) for each student in the class. Have students describe the relationships they can see. Ask: Given TV time, can you exactly predict reading time? Why? Why not?

**Relationship between time spent reading and time spent watching TV**




Draw out that although there appears to be a slight negative relationship—as TV hours go up, reading time tends to go down—you could not predict one exactly from the other because there is a lot of variation within this general trend. This individual variation introduces an element of uncertainty (or chance) into the process of prediction, even though there is an overall trend.

## What is Average?

When reading newspaper articles, books or information on the Internet, ask students to consider what is meant by the word 'average'. For example: If they read that the average life expectancy for a lion is 20 years, ask them to consider what this means. Does this mean that most lions will live for 20 years? (Link to Summarise and Represent Data, Key Understanding 6.)

## Skeletons

Have students examine the dot plot made in 'Skeletons' (see page 170), and make simple descriptive statements about the relationship between height and length from ankle to knee. Ask: Are there patterns in the placement of the pins on the dot plot? How do you know? Do you think leg length is related to height? Do the tallest people always have the longer lower leg?

# Later ✓✓✓

## Fractions

Invite students to use fractions or percentages to compare data in categories. For example, consider the table below:

| Girls' and boys' preferences for lemonade or orange juice | | |
|---|---|---|
| | Boys | Girls |
| Lemonade | 15 | 5 |
| Orange juice | 15 | 15 |

Ask: Could you use the numbers to say what drinks to order for the school camp? Could you use the numbers to say that the girls and boys liked orange juice equally as much? Do more girls out of the total number of girls like orange juice? Have students find the fraction of boys who like both drinks out of the total number of boys. Repeat this for the girls. Ask: How does changing the numbers into fractions change the results of comparing the two groups? Draw out that if there is the same number of girls as boys it is possible to compare the numbers as is, but if not, we need to convert to a percentage or fraction to judge.

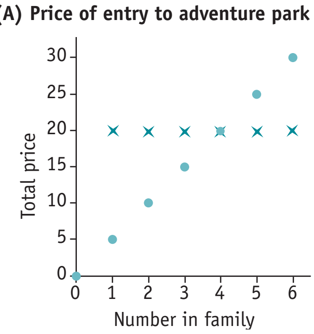
## Comparing Variation

Have students collect data from their class science investigations. For example: Does fertiliser make wheat plants grow taller? Have students plant seeds and measure the heights of at least 30 or 40 wheat plants regularly over a number of weeks. Only half the plants are fertilised. Have students plot all of the growth patterns and try to decide whether fertiliser makes a difference. Ask: What if some of the unfertilised plants grow taller than some of the fertilised plants? Does this mean the fertiliser makes no difference, or is there another way to compare our data? (See Sample Lesson 2, page 237.)

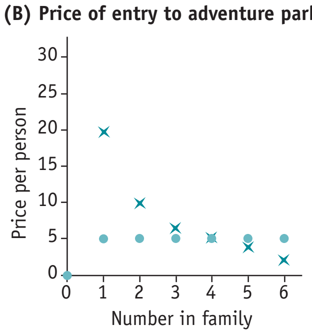
## Holidays

Present the two graphs below and the scenario of families on a trip to an adventure park. You can buy a family entry ticket for $20, or you can buy individual tickets for $5 each. Ask: Which graph shows which of the following? How can you tell?

**(A) Price of entry to adventure park**




**(B) Price of entry to adventure park**




- Cost per person if family ticket is bought, for different-sized families. (B)
- Cost per person if individual tickets are bought, for different-sized families. (B)
- Cost per family if family ticket is bought, for different-sized families. (A)
- Cost per family if individual tickets are bought, for different-sized families. (A)

# SAMPLE LESSON 2

**Sample Learning Activity**—Later, 'Comparing Variation', page 236

**Key Understanding:** When we analyse and interpret data we are deciding what it says and what it means. There is a difference between the data itself and what we think it means.

## Teaching Purpose

When my class of 11- and 12-year-olds interpreted some data that they had produced to investigate their television show preferences, I noticed that they considered small differences between the preferences of boys and girls as significant, and that they were happy to base conclusions on these small differences. I wanted them to think more carefully about differences that we might normally expect to find between groups, and differences that are unusual and therefore might really be significant.

## Action

The class chose to investigate the question: *Does fertiliser make wheat plants grow taller?* Showing an awareness of the need for fair testing, the students worked in pairs to grow two plants, one with and one without fertiliser. They planned to keep all other variables identical for both plants: light, depth of seed in soil, amount of water, soil type, and temperature. The plants were watered and their heights measured and recorded twice a week for five weeks. I asked the students to graph the results of height against time for each plant on a separate set of axes. To facilitate comparisons between plants, I suggested that all students use the same scales on the axes of their graphs.

***As I wanted to focus my students' attention on the interpretation of the data rather than the organisation or summarisation of the data, I needed to make these decisions for the students.***

## Drawing Out the Mathematics

I began by asking the students to compare the graphs of the growth of their two plants.

*The one with fertiliser ended up much taller,* said Stacey.

*Not for us,* countered Paul, *there's not much difference at all.*

*Well I reckon the fertiliser made ours grow worse. It's not as tall as the one without fertiliser,* observed James.

*Would you use the fertiliser if you were a farmer?* I asked.

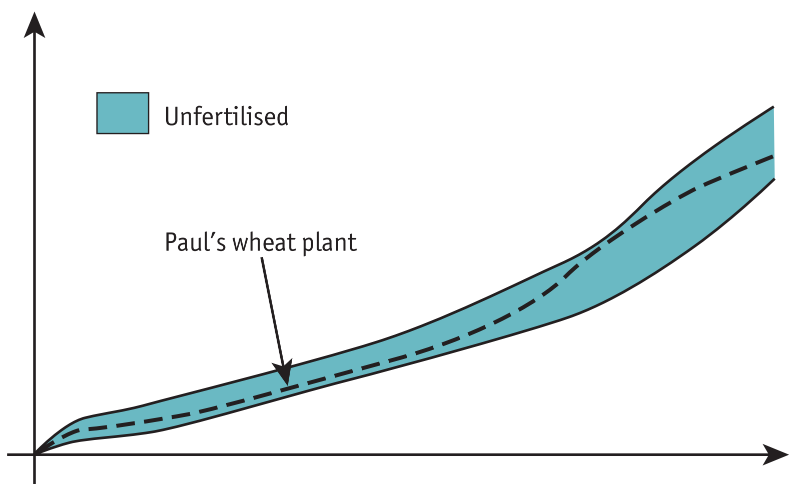
*No,* John answered. *If I was a farmer who had to pay for the fertiliser I'd want to be sure it was going to make a real difference.*

*But I think most of us found it made our plants grow more,* offered Petra.

*So how can we find out if the fertiliser does make a difference overall?* I asked.

The students saw the need to somehow combine the data of their individual plants before we could compare the fertilised and unfertilised plants. I suggested that, beginning with the unfertilised plants, we could plot the smallest and largest individual height measurement at each time point and so show the total variation in growth. By superimposing this transparency over their own graphs the students confirmed that, although the growth patterns and final heights of their individual plants varied greatly, the growth of all their unfertilised plants came within the range shown on the graph on the transparency.

***I used the same scales as the students on the axis for the transparency, so we could place it over the students' graphs and make comparisons.***




The students then superimposed the transparency over their individual graphs of the fertilised plants. They were surprised when many of these also fitted within the range of the unfertilised plants.

*See, our fertilised plant fits in that graph of all the unfertilised plants. I told you the fertiliser was no good,* remarked Paul.

*Well, our fertilised plant started off in the graph of all the unfertilised plants but then it gets taller near the end,* commented Jamie.

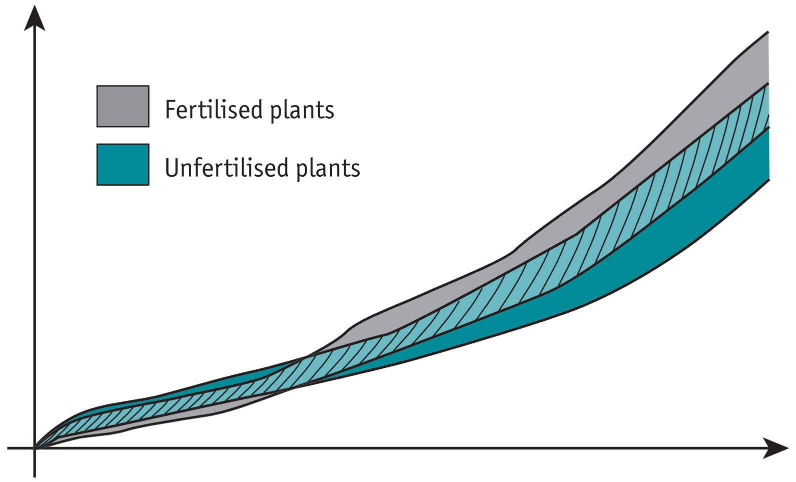
We then made a second overhead transparency showing the range of all the fertilised plants. We put one transparency on top of the other.

*They all grew about the same to start with,* noted Stacey.

*But they were still different,* said Kate.

*But it's sort of the 'same' different,* said Yvonne.

*After that the fertilised got 'more' different— they got taller but some were still the same,* said Joss.




Students struggled with the idea that although the variation **within** each group meant there was overlap (and you often couldn't tell just from one plant's growth whether or not it was fertilised), we could still say there was an overall difference **between** the two groups.

*So, even though my unfertilised plant grew taller than my fertilised plant, when you put them all together it doesn't show like that,* said James.

## Reflection

*So, do you think the fertiliser made a real difference, or do you think those plants might have grown like that anyway, without any fertiliser?* I asked the students.

*Joss:* It might have, some could have just grown taller.

*Stacey:* But they started the same and then only the fertilised ones were the highest—they were a lot more different at the end.

*Paul:* We might have just had flukey plants—they might have grown tall anyhow.

*John:* Why don't we do it again with more plants, so we can really tell?

I was satisfied that, when interpreting their data, they were beginning to think about what differences are due to normal variation and what differences are greater than normal.

# KEY UNDERSTANDING 3

***We need to evaluate the data we are using in order to be confident about the conclusions we have drawn.***

An important aspect of numeracy for everyday life is being able to judge the quality of the data and the inferential processes upon which we and others have reached conclusions. Students should begin to evaluate the quality and suitability of data (collection, organisation and display) for answering particular questions. During the primary years, this will mostly consist of reporting on their own data collection and handling, and reading and listening to the reports of their peers. For example, looking back over their own processes (or those of their peers) they should ask questions such as:

- When we refined our question did we still find out what we originally wanted to know? Is there anything we left out?
- How helpful were our measurements? Were they accurate and reliable? What of our survey questions—were they ambiguous or confusing, biased or 'leading'? What of our collection methods—were we careful or a bit sloppy? What effect might it have had on our data?
- Was the sample appropriate, that is, did we ask the right people, enough people? Did we collect enough cases, the helpful cases?
- Did we record all the information we needed to be able to answer our original question? What else did we need? Were the categories that we decided to use the right ones?
- Was our graph or table helpful? Did we need to redo it? What did we learn from that?
- Have we under- or over-interpreted our data, e.g. did we assume that two things being related meant that one caused the other?
- Can we answer our original question? If not, why not? What would we do differently next time?

By noting the strengths and weaknesses in their own and their peers' work, students should improve their data collection and handling over time. However, this does not mean that they should expect to be able to design an error-less data collection process and then simply implement it. The process of formulating and refining the question, producing data, summarising and representing it and then interpreting it is an iterative one, with the relationship between the data and the original question the focus of what is happening along the way. Each step may need to be reviewed and redone, indeed many experiments involve a trial (or 'pilot study') to see whether the processes set up will do the job.

Older students should begin to consider the credibility of data sources and to question the meaning and legitimacy of conclusions drawn from data that is not their own. Some of this data may be drawn from everyday contexts (such as advertisements on television or in children's magazines); other data may relate to topics in various parts of the school curriculum (e.g. a report on bicycle accidents from the Health Department, or a history database). Students should also be helped to reflect on the importance and use of their knowledge of chance and data in helping them form opinions and make decisions. Some of these opinions and decisions may be quite informal (e.g. forming views about famous people) and others more formal (e.g. undertaking research to decide what time of day school should begin).

## Progressing Through Key Understanding 3

Initially students make sensible comments about how well their collected data answers their questions. For example, after surveying fellow students about their food preferences they may say, *We thought asking people what food we liked would help plan the camp, but we didn't ask it very well and so we couldn't classify it. Next time we would ...* As students continue to progress they will, in addition, comment on the quality of their data collection and handling processes and suggest how they might improve them.

# SAMPLE LEARNING ACTIVITIES

## Beginning ✔

### Asking Questions

After students have collected information by asking people, have them check whether they have included everyone they should. Ask: Have you asked everyone? Should you ask the adults in the class? Are they included in this group?

### Yes/No

Hang cards with 'yes' on one side and 'no' on the other next to a list of students' names. Have students turn their card to the appropriate side to show whether they have completed work or need to do a particular thing. Display the question above the names, e.g. 'Who has had fruit?', 'Who needs to change their book?' Ask: Does the table help us to see who needs to change their book/have some fruit? (See Collect and Organise Data, Key Understanding 1.)

**Who needs to change their books?**

| Names | Yes/No |
|---|---|
| Sherry | Yes |
| Tom | No |
| Ali | Yes |
| | |

### Long Jump

When students draw conclusions from their own data, ask them to say what steps they took to make sure their data was accurate. For example: Have students use paper tapes to measure their long jumps, then compare tapes to see who jumped the furthest. Ask: How do we know our paper tapes really show us how far we jumped? Who can remember what we did to make sure we measured all of a jump, and no more? (Link to Collect and Organise Data, Key Understanding 4.)

### Popular Foods

Have students refer back to their initial question to explain what their displays of data show. For example: What is the most popular lunch food in our classroom? Ask students to watch what everyone eats for lunch and display the information. Ask: What does our graph tell us about the most popular foods? Invite students to consider whether the data they have collected could be generalised across different groups. For example: Ask them to collect data and create a table to show what food students in their class prefer. Ask: Would other children like the same foods? If we asked adults, would we get the same results? What if we asked teenagers?

### Pet Food

Encourage students to think about the source of data. For example: Have them look at a graph that shows the number of people who buy a certain pet food. Ask: Who could have drawn this graph? Where would their information come from? What are they trying to tell us?

### Scary Things

Have students make statements that link their data to their methods of collecting and sorting information. After reading scary stories such as *In the Middle of the Night* (Graham, 1989), ask students to decide whether to write or draw what scares them the most. Ask: Would it make a difference if we had sorted our pictures using different categories? If we collected and sorted what people wrote instead of these pictures would we have found out the same thing? (See Collect and Organise Data, Key Understandings 2 and 3.)

# SAMPLE LEARNING ACTIVITIES

## Middle ✔✔

### Comparing Data

When comparing collected measurement data, have students decide whether the results are accurate. Ask: Did we all measure in the same way? Do you think it matters? Why? Draw out that to be able to compare their data they need to be accurate, and that to get an accurate measure they all need to start and finish all their measurements at the same spot. (Link to Collect and Organise Data, Key Understanding 4.)

### Favourite Foods

While students are reporting on and discussing their survey data, draw attention to difficulties they have in reaching conclusions. For example: Students might ask about favourite fast food and have some people name a type of food (chips, Chinese) and others name a company. Ask: You are saying that people answered the question the wrong way—how could we avoid that if we did it again? Could it be that we asked the wrong questions? Perhaps our questions weren't clear?

### Computer Games

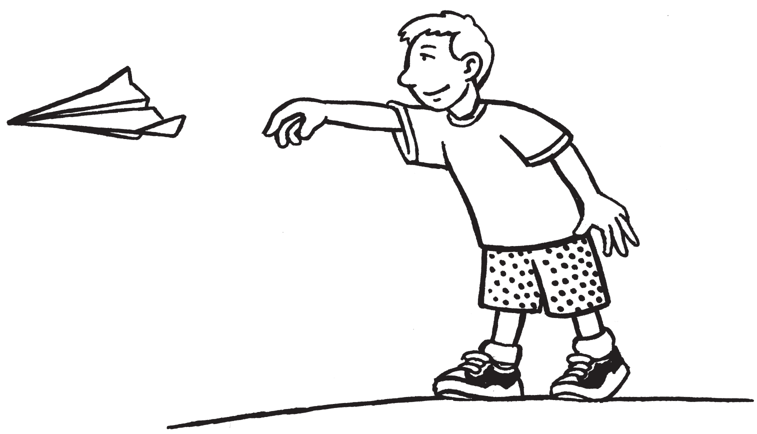
Invite students to review data they have collected and say whether it answers their original question. For example: Have students collect data to show which computer games children play. Ask: Does this data show us which game is the most popular? If not, why not? Was it the way that we collected the information, or the way that we organised the information? Did we collect the wrong information to answer our question?

### Recycling

Have students make predictions based on data and then say how sure they are of their conclusion. For example: Ask them to review a graph of the recycling habits of people in their community and say whether more or fewer items will be recycled in the future. Ask: Does the graph show a trend over time? Can we use this trend to make a prediction about the future recycling habits of the community?

## Paper Planes

After students have collected data to answer a question of interest, have them reflect on their data representation method. For example: Have them collect, then represent data on the best design for a paper plane. Ask: Was this the best way to represent the data? Would it help to draw a table/graph instead? Could we redraw/redo it to better show a relationship in the data?



## Eye Colour

Invite students to consider two different representations of the same data and say how they are the same and different. For example, compare the tables below:

| Name | Eye colour |
|---|---|
| Mai | blue |
| Freya | brown |
| Peter | blue |
| Kim | brown |
| etc. | |

| Name | Eye colour |
|---|---|
| Brown | /// |
| Green | // |
| Blue | / |

Ask: Are there more people with brown eyes or green eyes? Which table helps you to see this more easily? What colour eyes does Freya have? Which table tells you this? (Link to Summarise and Represent Data, Key Understanding 4.)

# SAMPLE LEARNING ACTIVITIES

## Later ✔✔✔

### Examining Data

When examining data obtained by their own survey, encourage students to think about the source of the information. Ask: Did we ask the right question? Should we have used fixed-choice instead of open-answer questions? Did we ask the right people, enough people? Draw out that the way the survey was conducted may affect the results. (Link to Collect and Organise Data, Key Understanding 2.)

### Who to Ask?

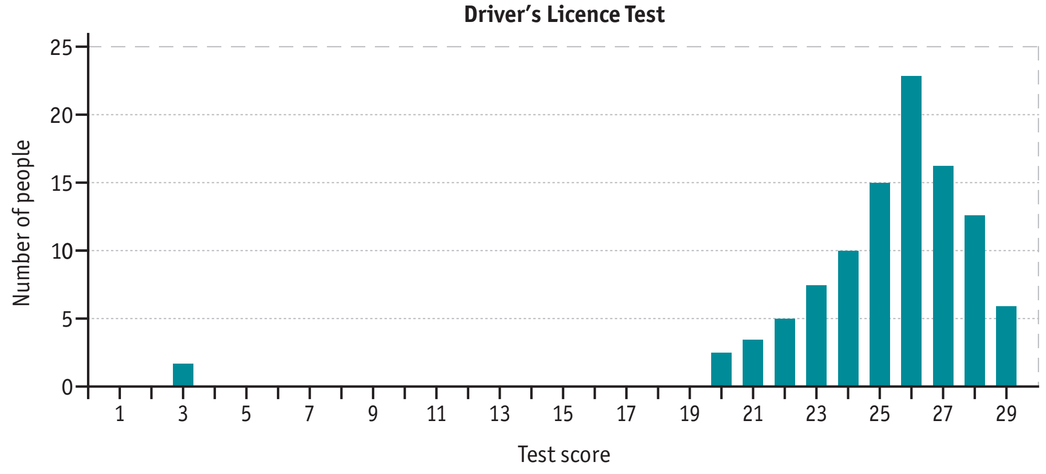
When examining data presented in newspapers, magazines, books or on the Internet, encourage students to think about the people who were asked in order to produce the information. Ask: Were these sensible people to ask? Is it reasonable to generalise from these people or has the newspaper (magazine, etc.) gone too far?

### What's Missing?

When students are reading newspaper articles, books or information on the Internet that use data to illustrate a point, ask them to consider whether any information is missing that may affect the data. Have students brainstorm questions they would like to ask the people who produced the data. For example: With data showing the average age of lions as 20 years, they might want to know whether the information was collected on zoo animals or wild animals, and whether zoo animals live longer than animals in the wild.

### Outliers

Present students with data that contains obvious outliers (see diagram below) and ask them to explain how this might have occurred. Focus on the fact that sometimes we make mistakes when collecting data, although sometimes these outliers are part of our data.



### Balloon Power (1)

Have students say whether the data they collected enabled them to answer their question/s. For example: Ask them to consider their data about the balloon-powered cars. Ask: Did the data help you answer the reworked questions you asked? Did it help you answer your original question? Would collecting different data help you to better answer your original question? Can you now build a car that travels further? What would you do differently next time? Why? (See Summarise and Represent Data, Key Understanding 5.)

### Balloon Power (2)

Extend the previous activity and discuss which type of data display helped them to see if there was a relationship between two measures. For example: Have students compare the ways they displayed the weight of each car with the distance travelled. Ask: How did you use the display to work out that there was or wasn't a relationship between the measures? (See Summarise and Represent Data, Key Understanding 5.)

### Favourite Drinks (1)

Encourage students to see how asking a question in a different way can give different results. Ask: Which do you prefer—water or lime cordial? Have them record the result. Now ask students to write down their favourite drink. Ask: How do the results compare? Draw out that results obtained from a multiple-choice and an open-ended question will produce different results. (Link to Collect and Organise, Key Understanding 2.)

### Favourite Drinks (2)

Extend the previous idea. When they are interpreting data, ask students to consider how a question was asked, by whom and in whose interests the survey was collected. Ask them to think about the following different conclusions:

- 8 out of 10 children prefer water to cordial
- For those who expressed a preference, 8 out of 10 children preferred water to cordial
- 8 out of 10 parents said their children prefer water to cordial.

Ask: What is different about these statements? What questions might have been asked in each case? What effect would the word 'lime' have in the question, rather than just 'cordial'? Why would it be in someone's interest to ask the question that way?

## Later ✔✔

### Relating Data

Have students review data that is being used to illustrate a particular point, e.g. a newspaper article on the effect of greenhouse gases on the atmosphere, or the rate of logging in the Amazon rainforest. Ask them to say how the data is related to the issue. Ask: Can the conclusion drawn in the article be made on the basis of the data presented? Can we be sure that one event actually causes the other?

### Tables and Graphs

During Technology and Enterprise or Society and Environment activities, when students have individually produced various tables and graphs to support their report, have them swap tables and graphs (without the report) with other students. They study the tables and graphs, and report what they think they can conclude from the information. Ask: How do these conclusions compare to the original report made by the designer?

### Misleading Representations

Invite students to discuss the effect of misleading representations of data on the way information is interpreted. Provide examples from newspapers, magazines and the Internet of bar graphs or histograms that can be misleading because of one or more of the following:

- the vertical axis does not start at zero
- the length or width of bars are not proportional to each other
- the horizontal axis is made much narrower
- the values of intervals are not equal.

Ask: What effect might each have on the way you interpret the information?

CHAPTER 7

# Markers of Progress

This chapter elaborates on how students progress through:

- **Understand Chance**
- **Collect and Process Data**
  - Collect and Organise Data
  - Summarise and Represent Data
- **Interpret Data**

Understand Chance

## Understand Chance

***Initially*** students recognise an element of chance in many things that are a part of their lives. They understand expressions such as 'will happen', 'won't happen', 'might happen', 'could happen' and 'couldn't happen' by responding to and using them. For example, they say: *Our new baby might be a girl or it might be a boy* or *It might rain tomorrow*. They show some understanding that repetitions of chance actions are likely to produce different results. For example, when choosing lucky dips, they know that they may get something different each time they 'dip'.

***As students continue to progress*** they begin to understand the idea of 'impossible' and to distinguish impossible things from those that are possible, even if unlikely, by thinking about the situation. They can tell it is impossible to get a cube-shaped jelly bean from a pack, because there isn't such a thing. They also distinguish impossible from unlikely events and say that it is not very likely that they will go to the pool after school, but it could happen.

Understand Chance

Students understand that of the possible outcomes for daily events, some are more likely than others. For example, on a warm and cloudless summer's day, they say that it is more likely that it won't rain than that it will. They also identify possible outcomes for events that are familiar or that they have observed, sometimes by collecting data. For example, they can say *I tried it out and found the hoop could go over no pegs, one peg or two pegs, but it couldn't fit over three because they are too far apart*.

Students may not necessarily list systematically all the possibilities of an experiment, but can list some that satisfy the given criteria. For example, given two or three ice-cream flavours, they may not identify all of the possible combinations for a double cone, but can list some of the possible combinations.

***Next***, students distinguish certain from uncertain things. They know that certain events include those that must happen and those that cannot happen, and uncertain events are those that may or may not happen. By comparing events within their personal experience, they also describe events as being more and less likely. For example, they say *It is more likely to rain in Bunbury in July than in January* and provide relevant reasons. They can order events from least likely to most likely and can justify their choice by referring to past or known information. For example, they say *I don't think my teacher will come to school tomorrow with green hair—it's almost impossible, but it is even less likely tomorrow will be Wednesday because today is Friday, so that's impossible*.

Students distinguish situations that involve equally likely events from those that do not. For example, they recognise that a spinner with four equal differently coloured sectors is equally likely to stop on any of the colours, but acknowledge that this is not true for another spinner that is coloured in four unequal sections. They can make informal statements about how one might influence the chance of an event happening. For example, they say *I am less likely to have an accident if I take the back road because I don't have to cross any busy streets*.

Students list all possibilities for straightforward situations, such as tossing two coins or dice, and with prompting can list all possibilities for a simple experiment. For example, they can list the possible ways of pairing up four socks in a washing basket.

***As students progress further*** they understand that the essential nature of chance processes means that things that are very unlikely are still possible and that things that are very likely may not happen. They can use the scale from 0 to 1 informally, placing everyday chance-related expressions such as 'impossible', 'poor chance', 'even chance', 'good chance' and 'certainty' on the scale. Unprompted, they list systematically all possible outcomes for a one-step experiment and use this information to work out numerical possibilities. Therefore they can list all the possibilities when tossing two dice and adding the result in order to determine which score has most likelihood of occurring.

Students use a range of sources of information to put things in order from least likely to most likely. For example, they use published data to order towns from the one most likely to have an earthquake to the one least likely. Given that red is the winning colour, they could order the following spinners from the one they'd rather have to the one they'd rather not.

They can design a probability device, such as a die, spinner or bag of coloured beads, to produce a specified order of probability. For example, they colour a spinner with eight segments so it would be most likely to stop on red, least likely to stop on green and have the same chance of stopping on yellow as blue.

***Later***, students understand the notion of the randomness of chance events. For example, by throwing a die 60 times, the numbers don't occur in order, and if a 6 is thrown, it is equally likely that a 6 will occur on the next throw. They understand that probability is the way in which chance is measured. They know that when something cannot happen, it is described as having a probability of 0; when something must happen it is described as having a probability of 1; and that the probability of all other things lies between 0 and 1. By listing a sample space for one-step events, students are able to allocate a numerical value to a probability. For example, if one-quarter of a spinner is red and the rest yellow, the probability of getting red on a spin is one-quarter and yellow is three-quarters.

Students represent probabilities as fractions, decimals and percentages and can move freely between these forms. They can rank events using fractional numbers, decimals or percentages. They use their understanding of equivalent fractions to judge whether events are equally likely. They can interpret expressions of probability in general usage, such as 'The probability of rain tomorrow in Perth is 30 per cent' and 'There's a fifty-fifty chance of rain tomorrow in Paris' to conclude that it is more likely to be raining in Paris tomorrow.

Students use the fact that the sum of the probabilities in any sample space is 1 to determine the relationship between an event and its complement. For example, if they determine that the probability of a spinner showing red was 0.3, they can conclude that the probability of not showing red was 0.7. They determine the probability of an event using the results of an experiment and can use this to predict the result of a repetition of the experiment. For example, if they throw a drawing pin 20 times, they can determine the probability of it landing 'point down' and use this to predict how many times a drawing pin would land 'point down' in 100 throws.

Students begin to explore the notion of simulations using chance equipment through games. For example, they can simulate the result of the quarter-finals in tennis by tossing a die to determine the winner of each game in each round to the finals. Tossing a 1, 2 or 3 could result in the higher ranking of the two players winning; 4 or 5 could result in the lower-ranked player winning and a 6 could be ignored and the die thrown again. They appreciate that repeating the simulation a number of times could increase the validity of the results.

# Collect and Process Data

## *Collect and Organise Data*

***Initially*** students participate in class discussions that draw out simple questions about objects or pictures. For example, when prompted to suggest what they could find out about the fruit they have brought to school, students might pose simple questions, such as *What type of fruit is there most of on the table?* or *Which fruit needs to be peeled before we can eat it?* They participate in class discussions about how they might find out the answers to these questions.

Students offer suggestions as to which objects they could collect or make to produce the data needed to answer simple questions posed by the teacher. For example, they select from a set of pictures the type of fruit they like best, and suggest how they can answer questions about their collections.

Students apply familiar and unambiguous criteria to classify and sequence data consistently. They can classify leaves according to length and/or colour and can then sequence the leaves according to length. They can also describe the likeness (or the difference) between several things. For example, *'The trees and the pencils are alike because they are long and round'*, or *'Some are leaves but some aren't'*.

***As students continue to progress*** they offer some appropriate data-oriented questions in class discussions. For example, having gone on a 'shape walk', they might ask what shapes occur most often in built things. They realise that they can answer some questions for themselves by collecting data. They can make predictions of what is likely to be shown by their questioning or surveys. For example, they think that the most popular type of fast food is pizza before they have carried out the survey.

Students participate in group discussions, suggesting ways of collecting objects or information and what data to collect for a survey. For example, they suggest asking each person in the class to draw a picture of her or his favourite fast food, and then collect the responses. They also offer suggestions about how to classify objects or information into categories they have created. For example, having asked class members what fast food they like most, they may classify foods such as noodles, sweet and sour, and satay under the heading 'Asian foods'. Students understand that once data is collected, some sorting or organising will be needed, and contribute suggestions on how to do this. They work in groups or pairs and follow a plan.

For example, they order the favourite fast foods from most to least popular by placing the collected pictures in piles and then sequencing the piles.

***Next***, students investigate some situations that extend beyond their immediate class group or families. For example, they suggest information to collect to answer particular questions, such as to describe the insect population in the school.

When prompted, students recognise the need to clarify and refine their questions to decide what data to collect, and attempt to do so. For example, they think about whether for 'most popular pet', they mean the most-liked pet or the most-common pet, and decide whether an aquarium of six fish counts as six votes for fish or one. They are also reasonably careful in their data collection. For example, in measuring arm lengths, they think about where to measure to and from and take care to do the same each time. In making tallies they are careful not to miss any responses.

Students think about how to organise data so that it is helpful for answering a particular question, such as which insects are most common. They suggest a suitable way to classify data and thus may sort insects into groups, or categories, such as beetles, butterflies and ants. Further, they may improve their descriptions of categories to clarify what the category includes or excludes so that when classifying insects into 'alive' and 'not alive', they have to decide where cocoons belong.

With prompting, students reorganise data to answer a new question, such as whether some insects occur more in certain parts of the school ground or at certain times of the day. They organise their data in tables and diagrams that show frequencies for different categories, using simple formats based on tallies or organised lists. For example, they show the number of insects seen in a two-way table, with insect types one way and parts of the school grounds the other way, or they may keep a tally of the results of tossing a die 60 times. Students can record data at regular time intervals, e.g. example they record lengths at half-hourly intervals to show the change on length of a shadow.

***As students progress further*** they investigate a wide range of practical problems that may not be obviously mathematical, but for which mathematics might help. They suggest what data is to be collected to help estimate numbers or quantities. For example, in planning an apple-eating competition for the school fete, they organise the overall task into a series of sub-tasks, some of which require mathematics. As one part of their preparation, they may need to decide how high to hang the apples and how long to cut the strings. They do not solely rely on guessing or opinion but, unprompted, will think of collecting some data.

Students construct and use their own categories to answer specific questions. Therefore, for a project on animals in their area, they ask how different animals move and decide to classify animals by their ability to walk, fly, wriggle or swim. They also suggest indirect ways, such as using a database or reworking existing data when direct data are unavailable. They are aware that different classifications may be necessary to answer different questions and can suggest how to improve a classification strategy to better suit the purpose. They attempt to reframe a simple survey question to make it less ambiguous, or to make responses easier to interpret. They begin with 'Which of these colours do you like?' and, after trialling, revise to 'Which pair of colours would you like best for our logo?' Or they revise a survey question so it can be answered by 'yes/no' or a simple multiple choice.

They are prepared to experiment with how best to organise data into a form that is helpful for answering particular questions, and will vary their classifications to answer different questions.

Students design a test of their predictions about a probability device they have designed, such as a spinner that will come up red most often. They can also devise methods to collect and record data to show how data changes with time. Thus they choose half-hourly intervals when collecting data to show how the length of a shadow changes with time or how much water is collected from a dripping tap at hourly intervals.

***Later***, students work in small groups to survey people and collect data. For example, they sample people about their musical preferences or take measurements of their pulse rate after exercise. In doing this, they collaborate to develop and trial short sets of questions and plan how to collect and record accurate measurements or frequency counts. They also consider refining the processes of gathering data, from clarifying terms to improving the accuracy and consistency by asking the planned questions in the same way, measuring the same body part to the same degree of accuracy, and so on.

Students use some systematic strategies for organising their data, including grouping them into class intervals and entering them into databases, although they may need help with the classes or fields. They check and refine their recording of data by making and refining data collection sheets involving lists, tables or scales. They differentiate between data collected from a sample to data collected from an entire population and can choose a fair sample from a given population.

## *Summarise and Represent Data*

***Initially*** students group collections of objects either physically or by using pictures. They compare the size of their classified groups by lining up collected objects into rows to compare the groups directly, by using one-to-one correspondence, or by counting how many are in each group and comparing the numbers. They explain: *At first we thought there were about the same number of pears and strawberries, but we lined them up and there were more strawberries, or There are eight strawberries, seven pears and four bunches of grapes*.

Students work with other children or an adult to classify things they have collected, if the criteria are familiar. For example, they could sort clothes into 'summer' and 'winter' groups, if the distinctions are reasonably straightforward. They also sequence small collections of things. For example, placing pictures of family members or friends in order from the youngest to the oldest.

Students display their data and the results of their thinking by showing their collections physically or by drawing pictures of what they have done. For example, they can draw a picture of the 'graph' they made with actual pieces of fruit, or make a physical block graph using interlocking cubes with a different colour to represent walking, catching a bus or riding in a car to show how they came to school.

***As students continue to progress*** they use a variety of ways to summarise and display what they have found. For example, they use coloured buttons to represent children's snack preferences and arrange buttons in one-to-one correspondence, or draw pictures to represent each snack and underneath record the names of children who preferred this snack. They count accurately and efficiently to describe how many there are in a number of different categories. They also use organised lists or one-way tables to arrange information. For example, they draw a picture of each fast food type, list underneath the names of the people preferring it and count how many names are in each list.

Students make graphs and plots using one-to-one correspondence between 'real' data and a representation. For example, they draw a picture of the fast food types and ask the class to line up in front of the pictures, choose a particular colour counter for each snack type and use the counters to make block graphs, or produce a simple block graph by colouring in one square for each person who made a particular choice. They understand the need for a baseline and space blocks regularly (in provided grids) to allow comparisons to be made. They place direct measurement data in sensible sequences using the baseline. For example, cutting paper strips to fit around their heads, writing names on the tapes and making a bar (column) graph by lining up the bottoms of strips. They also compare heights (or lengths) of the columns in a block graph to place categories in order. For example, they say *This shows more children have a digital watch than one with hands* and *The fewest have no watch*.

***Next***, students understand that lengths in bar graphs can be used to represent measurements they have made at equal intervals over a period of time, such as the weekly mass of the class's baby guinea pig or the length of a shadow at hourly intervals. The lengths may also represent other measures such as time. For example, they mark paper strips to show the 24 hours of the day, shade the time between going to bed and getting up, cut shaded parts and make a graph of the times in bed. In their graphs, they use squares or pictures to represent more than one unit. For example, one square might be coloured for each five children.

Students report numerically on the results of making conventional tallies. They may use a conventional tally method to record the number of times a thumb tack falls on its side or on its top, and summarise the results by counting the tally for each location. They use Venn diagrams involving overlapping categories and can place information into the correct location in simple two-way tables. Thus they place name cards in appropriate sections of their diagram or table. They also summarise data in diagrams and tables that show frequencies for different categories. For example, the category may be 'type of food' and frequency may be 'the number of children' who chose that type. The names of children recorded in a Venn diagram may be replaced by the count of how many were in that category.

They use simple scales and labels on the axes of graphs. For example, they can produce (vertical and horizontal) bar graphs from frequency data, where one axis shows the whole numbers (0, 1, 2, 3, ...).

***As students progress further*** they make working graphs in order to explore data. For example, they may use sticky notes with food choices written on them to make quick block graphs based on different ways of classifying the foods. They understand that it is sometimes helpful to group data and, with assistance, can group data involving whole numbers into class intervals. For example, having estimated the number of sweets in a jar, students can organise the estimates in intervals such as 41–45, 46–50, 51–55, 56–60, in order to compare estimates with the true amount. They appreciate that grouped measurement data may be treated as categories.

Students display data in bar graphs where the axis is labelled with discrete categories including separate numbers, such as 25, 30, 35, ..., multiples such as 0, 5, 10, 15, ..., or they can group data into intervals, such as 21–25, 26–30, 31–35. They can extend the interval boundaries by half a unit either side to ensure that there are no gaps in the resulting histogram. Given a horizontal axis showing the progression of time, they can produce a graph using the vertical scale to help them plot data points. They can also represent data, including grouped data, using Venn diagrams and two-way tables with confidence and can construct tree (arrow) diagrams.

Students find the mean where there is sufficient data to make summarising sensible, such as *the average height of the girls in class is 149 cm*. They can use a mean to get an estimate of a number. For example, they count the number of sultanas in 25 boxes to estimate the mean number of sultanas in a box. They may use fractions to summarise data, saying that about  of the children said that they like to play tennis; or about  of Year 2 students ride to school,   walk, and the rest come by car. They also examine the spread of the data by determining the lowest, highest and middle ten scores and use these, together with the mean, to summarise what their data show. For example, while the mean distance around heads was 55 cm, they varied from 49 cm to 60 cm. They could suggest why average head size may not be helpful for designing hats.

***Later***, students display and summarise data in tables with provided class intervals and with frequencies. They sort data using a calculator or computer to compare groups. They use appropriate graphs, including circle graphs, and summary statistics to represent such data, generating some graphs with the assistance of a graphic calculator or computer.

Students use fractions and percentages to describe and compare their results. For example, in a survey, 32 out of 40 parents and 35 out of 60 students agreed with the statement ‘Teenagers should earn their pocket money’.  They convert each proportion to a percentage to make comparison easier.

Students use the measures of central tendency—mean, mode and median—to summarise data. They can perform the calculations in written form or by using a calculator or computer. They understand the advantages and disadvantages of these ‘averages’ and can determine which is most appropriate to a given set of data.

Students use a range of graphs such as line plots and scatter plots for bi-variate data and dot frequency plots, stem (stem and leaf) plots, bar graphs, compound column graphs and histograms for uni-variate data. They sketch graphs informally that give a 'feel for' how familiar things change over time without recourse to careful data collection or point plotting.

Students display information in tables involving provided class intervals and can use quite complex scales on axes to produce the full range of graphs required. They plot available time data using more complex scales, where each time may not be represented on the horizontal axis. They can represent grouped uni-variate data in frequency histograms.

## Interpret Data

***Initially*** students reach simple conclusions based on counting and one-to-one correspondence. For example, having counted eight strawberries, seven pears and four bunches of grapes, they say There are more strawberries, and hence draw the conclusion that strawberries are the fruit brought to school by most children.

Students interpret results collaboratively collected in a table. For example, they conclude that *There are more dogs shown in our table, so more people have dogs as pets*.

***As students continue to progress*** they not only describe what their own data collection shows, but also read and listen to other students' reports. These reports can be oral or written. For example, they say *Delia's graph shows that most people she asked liked chicken best, but almost as many liked pizza* and *That isn't the same as I found, but it is pretty close*, or they may write a few sentences to describe the results of their data collection.

Students extract frequencies from lists and one-way tables and use them to compare frequencies. For example, they say *This shows that four people like apples and six like bananas*. They can interpret block graphs produced by others. For example, they say *Their graph shows that most children they asked like pasta better than pizza*. They also describe how their graph shows the results of their data collection. For example, they say *We spun the spinner ten times. These three squares on our graph show the three times we got yellow, these (pointing) show we got red six times and green once. Blue didn't come up*.

***Next***, students interpret tables, diagrams, bar graphs and pictographs produced by themselves and others, including their peers, drawing sensible conclusions. They can read a tally, extract data from simple one- and two-way tables and determine frequencies from a pictograph or a bar graph in which each unit is marked on the axis. For example, they say *This shows that of the 17 berries, ten were green and edible, seven were green and not edible or Their graph shows that 11 of the people asked said they preferred frozen yoghurt*.

Students explain what their displays show and comment on their predictions in the light of their collected data. For example, they say *We thought there would be more butterflies in the vegetable garden than near the roses, but there weren't* and *We thought that the average number of black jelly beans in a packet would come out least and we were right*.

***As students progress further*** they make sensible comments about how well their collected data answer their original questions. For example, after surveying fellow students about their food preferences, they say *We thought asking people what food they liked would help us to plan the camp, but we didn't ask it very well and so we couldn't classify it. Next time we would …*

Students describe information from diagrams, including tree (arrow) diagrams, Venn diagrams or Carroll diagrams. For example, they say *This shows that six people like pizza but not hamburgers, eight like hamburgers but not pizza, two don't like either and 14 like both*. They can read the information provided on axes of bar and line graphs, including where all calibrations on the scale may not be labelled. They read frequency and other types of information from a range of tables, histograms and bar graphs (including instances in which data have been grouped). For example, waist measurements may be grouped into intervals to reflect tracksuit pants sizes. They can interpret and report on information provided in line graphs, describing trends in the data informally. For example, *This shows that we raised more money each week until the last two weeks when …* They also interpret fractions representing proportions from a set of data and means, commenting informally on their findings.

***Later***, students interpret a variety of graphs, including frequency histograms for grouped data, where the scales on the axes must be read between calibrations.

Students describe the results of their data collection, talking informally about relationships they see in the data. For example, they may note that faster readers also seem to read the most, but cannot state that one causes the other. They also comment on the quality of their data collection and handling processes, and suggest how these might be improved both before and after interpretation. By checking the accuracy of the data, students can refine their procedures and improve the accuracy of the data collected.

Students distinguish between different 'averages' in their interpretation of data. They can state: *The mean height of the students in the class has increased over the semester and the median height hasn't changed. This would probably mean that half the class hasn't grown any taller but that some of the taller students have grown*. They are able to extract information from a variety of descriptions, tables, diagrams and graphs produced by others, including informal graphs showing the relationships between two quantities.

Students present written or oral reports on the information gathered in their surveys, describing the questions, data collection and conclusions, and commenting on the scope for improvement. They can also present reports on the information displayed in a range of tables and graphs.

## Classroom Plan for Week ___, Term ___ Year Level: ___

| Outcome/Key Understanding | Mathematical Focus | Activities | Focus Questions | Observations/Anecdotes |
|---|---|---|---|---|
| | | | | |

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